Name: Solutions

11U6 W24 - Exponential Functions Practice Test

Multiple Choice

Circle (CLEARLY) the choice that best answers the question, ***AND*** *write the letter of your choice beside the question.*

1. Which of the following is equivalent to 1?

B

a. $1^3 + 1^2$ $= 2$

b. $5^{-4} \times 5^4$ $= 5^{-4+4} = 5^0 = 1$

c. $(-1)^3$ $= -1$

d. $\dfrac{13^4}{13^{-4}}$ $= 13^8$

2. Which of the following is equivalent to the expression $12^{-4} \times \dfrac{12^2}{\left(12^3\right)^{-1}}$? $= 12^{-4+2-(-3)}$ $= 12^1 = 12$

D

a. $\dfrac{1}{12^4}$

b. $\dfrac{1}{12^9}$

c. 12^9

d. 12

3. What is $\sqrt[3]{-125^4}$ in exponent form? $\left(-125^4\right)^{\frac{1}{3}} = (-125)^{4/3}$

D

a. $(-5)^{\frac{4}{3}}$

b. $125^{-\frac{3}{4}}$

c. $(-125)^{\frac{3}{4}}$

d. $(-125)^{\frac{4}{3}}$

4. Which function describes exponential growth?

C

a. $f(x) = 180(0.95)^x$ decay

b. $f(x) = 13.7\left(\dfrac{1}{5}\right)^x$ decay

c. $f(x) = 18.9(10)^{\frac{x}{4}}$ growth — "base" is bigger than 1

d. $f(x) = -24(-6)^x$ not exponential (bases are positive)

Written Solutions: Provide clear solutions to the following problems. You can recieve up to **3 Communication points** for how well you present your mathematics.

5. Evaluate. No decimals are allowed.

a) $\left(\frac{2^3 \times 3^2}{2^5}\right)^{-2}$

$= \left(2^{3-5} \cdot 3^2\right)^{-2}$

$= \left(2^{-2} \cdot 3^2\right)^{-2}$

$= 2^4 \cdot 3^{-4}$

$= \frac{2^4}{3^4} = \frac{16}{81}$

b) $\sqrt[7]{128^5}$

$= \left(128^5\right)^{\frac{1}{7}}$

$= \left(\left(128\right)^{\frac{1}{7}}\right)^5$

$= (2)^5$

$= 32$

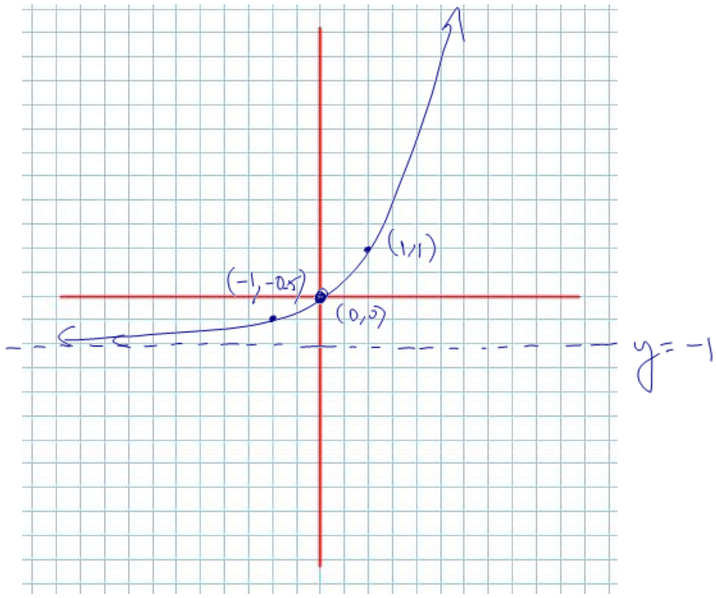
6. Simplify the expression. Express your answer with positive exponents.

$\frac{\left(81a^{16}b^{-4}\right)^{\frac{1}{4}}}{\left(4ab^3\right)^3} = \frac{81^{\frac{1}{4}} \cdot a^{16\cdot\frac{1}{4}} \cdot b^{-4\left(\frac{1}{4}\right)}}{4^3 a^3 \left(b^3\right)^3} = \frac{3a^4 b^{-1}}{64 a^3 b^9}$

$= \frac{3a^{4-3} b^{-1-9}}{64}$

$= \frac{3a}{64b^{10}}$

7. Sketch the function $g(x) = 2^x - 1$. Determine the y-intercept, the horizontal asymptote and whether the function describes growth or decay.



horiz. asymptote: $y = -1$

base $= 2 \Rightarrow$ exponential growth.

y int $\Rightarrow x = 0$

$g(0) = 2^0 - 1 = 1 - 1 = 0$

$\Rightarrow (0,0)$

Two other points.

x	$g(x) = 2^x - 1$
-1	0.5
1	$1.$

8. A delicious apple pie was left to cool in a room whose temperature was 21°C. The temperature changes according to the function $T(t) = 168\left(\frac{1}{2}\right)^{\frac{t}{30}} + 21$, where t is in minutes and $T(t)$ is temperature in Celcius. Determine the temperature of the pie after:
i) 30 minutes, ii) 2 hours and 20 minutes

Explain the ***meaning*** of the y-intercept and the horizontal asymptote in the context of this problem.

units must match with "t" units. (min)

i) $T(30) = 168\left(\frac{1}{2}\right)^{\frac{30}{30}} + 21$

$= 105°C$

ii) 2 hours = 120 min
2 hours' 20 min = 160°

$T(120) = 168\left(\frac{1}{2}^{\frac{160}{30}}\right) + 21$

$= 25.2°C.$

The y int (0, 189) contains the original temperature 189° C.

The horizontal asymptote $\boxed{y = 21}$ is the ambient room temperature

9. The value of a piece of art appreciates after it is purchased according to the formula

$V(t) = 75500(1.013)^t$

where $V(t)$ is the piece of art's value after t years. Determine:

a) The purchase price of the artwork
b) The rate of appreciation *(the rate of growth in value)*
c) The piece of art's value after 5 years.

a) Purchased when $t=0 \Rightarrow V(0) = 75\,500(1.013)^0$
$= \$75\,500$

b) $1.013 = 1+r$
$\Rightarrow r = 1.013 - 1$
$= 0.013$
$= 1.3\%$

c) $V(5) = 75500(1.013)^5$
$= \$80\,536.76$
$\therefore$ after 5 years the art is worth $80536.76

$P(t) = P_0(1+r)^t$ $\quad r = 0.035$

10. A small town, with a population of 8500 (P_0) in 2000 experiences a poulation growth rate of 3.5% per year every year after 2000.
a) Develop an equation which models the poulation of the town t years after 2000 (thatis, we take the year 2000 to be $t = 0$).
b) Determine the population of the town in 2016.
c) In what year will the population double?

a) $P(t) = 8500(1.035)^t$

b) $2016 \Rightarrow t = 16$
$P(16) = 8500(1.035)^{16} = 14\,738$ people.

c) want "t" for $P(t) = 2B = 17\,000$
$\Rightarrow 17000 = 8500(1.035)^t$

$\div 8500 \Rightarrow 2 = 1.035^t$

log both sides $\log(2) = t \cdot \log(1.035)$

$\Rightarrow t = \dfrac{\log(2)}{\log(1.035)}$

$\doteq 20$ years

$\therefore$ In the year 2020 the population will be doubled.