MCR3U U4 Extra Review Problems - Exponential Functions

Learning Goal: The goal of this problem set is to help you strengthen your learning in Exponential Functions.

Success Criteria:
I can use the exponent laws
I can recognize that a rational exponent has two parts: a traditional exponent and a root.
I can use the general formulas for Growth and Decay (Growth's power has a base greater than one, Decay's power has a base less than one).

1. Evaluate. Express your answer in rational form.

$(-3)^{-4}$

$= \frac{1}{(-3)^4} = \frac{1}{81}$

2. Evaluate. Express your answer in rational form.

$7^{-3} \div \left(\frac{7^2}{7^{-1}}\right)^{-2}$

$= 7^{-3} \div (7^3)^{-2}$

$= 7^{-3} \div 7^{-6}$

$= 7^{-3-(-6)} = 7^3 = 343$

3. Simplify the expression. Express your answer with positive exponents. Explain each of your steps.

$\frac{m^{-4}n^{-6}}{mn^{-1}}$

$= m^{-5}n^{-5}$

$= \frac{1}{m^5n^5}$

4. Simplify the expression. Express your answer with positive exponents.

$\left(\frac{(4x^6)^3(4y^{-8})}{(2x)^4(12y^3)^2}\right)^{\frac{1}{2}}$

$= \left(\frac{4^3x^{18}\cdot 4y^{-8}}{2^4x^4(12^2y^6)}\right)^{\frac{1}{2}}$

$= \left(\frac{4^4x^{14}y^{-14}}{(2^4)(12^2)}\right)^{\frac{1}{2}}$

$= \left(\frac{4^2x^7y^{-7}}{(2^2)(12)}\right)$

$= \frac{16x^7}{48y^7}$

$= \frac{x^7}{3y^7}$

5. Simplify the expression. Express your answer with positive exponents.

$$\frac{\sqrt{81z^{16}}}{\sqrt{100z^{-4}}}$$

$$\frac{(81z^{16})^{\frac{1}{2}}}{(100z^{-4})^{\frac{1}{2}}}$$

$$\frac{9z^{8}}{10z^{-2}} = \frac{9z^{8-(-2)}}{10} = \frac{9z^{10}}{10}$$

6. Simplify the expression. Express your answer with positive exponents.

$$\left(\frac{(x^{12})^{0.25}(216x^{9})}{(3x)^{6}(x^{18})^{0.5}}\right)^{-\frac{1}{3}}$$

$$\left(\frac{(x^{3})(216x^{9})}{3^{6}x^{6}(x^{9})}\right)^{-\frac{1}{3}} = \left(\frac{216x^{12}}{3^{6}x^{15}}\right)^{-\frac{1}{3}} = \left(\frac{3^{6}\cdot x^{15}}{216x^{12}}\right)^{+\frac{1}{3}} = \frac{3^{2}x^{5}}{6x^{4}} = \frac{9x}{6} = \frac{3x}{2}$$

7. The function $g(x) = -2(3^{2x-6}) + 9$ is the result of transformations of $f(x) = 3^{x}$. State the y-intercept, the horizontal shift and the horizontal asymptote.

FACTOR $(2(x-3))$

y_{int}: $g(0) = -2(3^{-6}) + 9$
$= 8.997$

Horizontal Shift: 3 right

HORIZONTAL ASYMPTOTE: $y = 9$

8. In 2001, a sum of \$4000 is invested at a rate of 6.5% per year for 5 years. What is the value of the investment when it matures?

$A(t) = A_0(1+r)^{t}$, $A_0 = 4000$, $r = 0.065$, $t = 5$

$A(5) = 4000(1.065)^{5}$

$= \$5480.35$

9. A cup of hot liquid was left to cool in a room whose temperature was 18°C. The temperature changes according to the function $T(t) = 80\left(\frac{1}{2}\right)^{\frac{t}{30}} + 18$, where t is in minutes. Determine the temperature of the liquid after:

i) 30 minutes, ii) 60 minutes, iii) 5 hours

5 hours = 5×60 = 300 min

i) $T(30) = 80\left(\frac{1}{2}\right)^{\frac{30}{30}} + 18 = 58°C$

ii) $T(60) = 80\left(\frac{1}{2}\right)^{\frac{60}{30}} + 18 = 38°C$

iii) $T(300) = 80\left(\frac{1}{2}\right)^{\frac{300}{30}} + 18 = 18.08°C$

10. A baker records the internal temperature of a pie that has been left to cool on a counter. The room temperature is 14°C. An equation that models this situation is $T(t) = 68(0.5)^{\frac{t}{10}} + 14$ where T is the temperature in degrees Celsius and t is the time in minutes. Determine the temperature, to the nearest degree, of the pie after 15 minutes. How much time did it take for the pie to reach an internal temperature of 31°C?

$T(15) = 68(0.5)^{15/10} + 14$

$= 38°C$

$31 = 68(0.5)^{t/10} + 14$

$\Rightarrow 17 = 68(0.5)^{t/10}$

$\Rightarrow 0.25 = 0.5^{t/10}$

$\Rightarrow (0.5)^2 = (0.5)^{t/10} \Rightarrow 2 = \frac{t}{10}$

$\Rightarrow t = 20$ min

with the same base "power" = "power" MEANS the exponents are equal

11. A laptop computer loses its value each month after it is purchased. Its value as a function of time, in months, is modelled by $V(m) = 1800(0.92)^m$.

a) What is the rate of depreciation?

b) What is the value of the laptop after 3 months?

c) In which month after it is purchased does the laptop's worth fall below $1000?

a) General eqn: $V(m) = V_0(1-r)^m \Rightarrow 0.92 = 1-r$

$\Rightarrow r = 0.08 \Rightarrow r = 8\%$

b) $V(3) = 1800(0.92)^3 = \$1401.63$

c) $1000 = 1800(0.92)^m$

$\Rightarrow 0.\dot{5} = 0.92^m$ → guess and check: $m=7$ → $0.92^7 = 0.55$ (2 digits of accuracy)

→ approximately 7 months

↳ logs: $\log(0.\dot{5}) = m \cdot \log(0.92)$

$\Rightarrow m = \frac{\log(0.\dot{5})}{\log(0.92)} \doteq 7.04 \Rightarrow 7$ months.

12. The population of a town has grown at an annual rate of approximately 2.7%.
Write an exponential function which will describe the town's population over time.
How long will it take for its population of 18 450 people to double at this growth rate? Explain how you found your answer.

$P_0 = 18450$
$r = 0.027$

$P(t) = 18450(1.027)^t$

we want $P(t) = 36900$ (2×18450)

$\Rightarrow 36900 = 18450(1.027)^t$

$\Rightarrow 2 = 1.027^t$

① logs

$\Rightarrow \log(2) = t \cdot \log(1.027)$

$\Rightarrow t = \dfrac{\log(2)}{\log(1.027)} \doteq 26.$

② Guess and check $t = 26$

$1.027^{26} = 1.999 \sim 2$

$\Rightarrow$ 26 years. (I'm a good guesser)

13. Determine the exponent that makes the equation true.

$$13^x = \frac{1}{2197}$$

$\Rightarrow 13^x = (2197)^{-1}$

$\Rightarrow 13^x = (13^3)^{-1}$

$\Rightarrow 13^x = 13^{-3} \Rightarrow x = -3$

14 Simplify.

$$\left(\frac{r^3 s^{-1}}{r^2 s^6}\right)^{-1} = \left(\frac{r}{s^7}\right)^{-1} = \frac{s^7}{r}$$

15. Solve for x.

$$\sqrt[3]{\frac{1}{64}} + \left(\frac{243}{x}\right)^{\frac{2}{5}} = \sqrt{\frac{25}{4}}$$

$$\Rightarrow \frac{1}{4} + \frac{\left(243^{\frac{1}{5}}\right)^2}{x^{2/5}} = \frac{5}{2}$$

$$\Rightarrow \frac{3^2}{x^{2/5}} = \frac{10}{4} - \frac{1}{4}$$

$$\Rightarrow \frac{9}{x^{2/5}} = \frac{9}{4} \Rightarrow x^{2/5} = 4$$

$$\Rightarrow x = 4^{5/2} \quad \left(= \left(4^{\frac{1}{2}}\right)^5 = (2)^5 = 32\right)$$

$$= 32$$

16. The doubling time for a certain type of cell is 4 h. The number of cells after t hours is described by $N(t) = N_0 2^{\frac{t}{4}}$ where N_0 is the initial population.

growth!

a) Determine the number of cells after 24 hours if there were 3 cells initially.

b) What are the domain and range of this function in the context of this problem?

a) $N(24) = 3\left(2^{24/4}\right) = 3\left(2^6\right) = 192$ cells

b) Domain : $\{t \in \mathbb{R} \mid t \geq 0\}$ (no negative time)

Range : $\{N(t) \in \mathbb{N} \mid N(t) \geq 3\}$ the population grows from here

the number of cells can't be a fraction or decimal

17. An antique painting is purchased in 1980 for \$995. The value increases by 3.1% every year.

$1980 \Rightarrow t = 0$ $\qquad A_0 = 995$ $\qquad r = 0.031$

a) Write an equation that models the value of the painting after t years.

b) Determine the increase in value of the painting in the 6th year after it was purchased (from year 5 to year 6).

c) Determine the increase in value of the painting in the 25th year after it was purchased.

a) $A(t) = 995(1.031)^t$

b) Increase in 6th year $= A(6) - A(5) = 995(1.031)^6 - 995(1.031)^5 = \$35.93$

c) Increase in 25th year $= A(25) - A(24) = 995(1.031)^{25} - 995(1.031)^{24} = \$64.18$