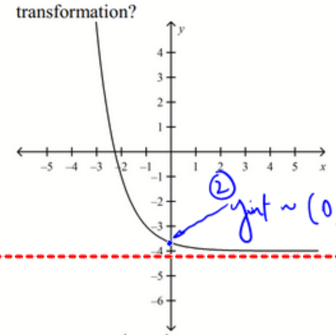
A∞Ω MATH@TD

Solutions

MCR3U U4 - Exponential Functions: More Practice for the test

1. The graph shown represents a transformation of the function $f(x) = 3^x$. What is the equation for the transformation?



② y-int ~ (0, -3.75) ✓ yes!

$y = -4$ horizontal asymptote ①

$\Rightarrow$ "$3^x - 4$" $\Rightarrow$ not a) or d)

a) $g(x) = -\left(3^{x-1}\right) + 4$

b. $g(x) = 3^{-x-1} - 4$

c. $g(x) = 3^x - 4$

d. $g(x) = 3^{-x-4}$

③ Compare y-int: $g(0) = 3^{0-1} - 4$

$= \frac{1}{3} - 4 \doteq -3.7$

③ Compare y-int

$g(0) = 3^0 - 4$

$= 1 - 4$

$= -3$

not correct $\Rightarrow$ only b) is possible.

2. Evaluate. Express your answer in rational form. (as a fraction.)

$$\frac{4^3}{16^0} \times \left(5^2 \times 5^{-3}\right)^{-2}$$

$$= \frac{4^3}{1} \times \left(5^{-1}\right)^{-2}$$

$2 + (-3)$

anything to the zero is 1.

$(-1) \times (-2)$

$$= 4^3 \times 5^{+2}$$

$$= (64)(25)$$

$$= 1600$$

3. Simplify the expression. Express your answer with positive exponents.

$$\left(\frac{\left(a^5 b^{-2}\right)^{-3}\left(a^6 b^7\right)^{-1}}{\left(a^{-7} b^{-8}\right)^{-4}}\right)^2$$

$$= \left(\frac{\left(a^{-15} b^{6}\right)\left(a^{-6} b^{-7}\right)}{a^{28} b^{32}}\right)^2$$

$$= \left(a^{-15+(-6)-28}\, b^{6+(-7)-32}\right)^2$$

$$= \left(a^{-49}\, b^{-33}\right)^2$$

$$= a^{-98} b^{-66}$$

$$= \frac{1}{a^{98} b^{66}}$$

4. State all transformations applied to "construct" the function $y = -3\left(2^{x+6}\right) - 1$. Also state the parent function and the domain and range.

	vertical	horizontal
FLIP	Yes	No
STRETCH	$\times 3$	$\times 1$
SHIFT	down 1	6 left

: "BASE 2"

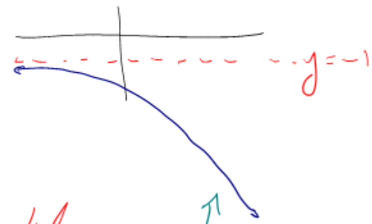
Parent: $f(x) = 2^x$

$D_y : \{x \in \mathbb{R}\}$ ← true of all exponentials

$R_y : \{y \in \mathbb{R} \mid y < -1\}$

$y = -1$ is the HA.

Sketch



flipped vertically

5. A barista records the internal temperature of a cup of coffee that has been left to cool on a counter. The room temperature is 18°C. An equation that models this situation is $T(t) = 80(0.5)^{\frac{t}{15}} + 18$ where T is the internal temperature in degrees Celsius and t is the time in minutes.
a) Determine the temperature, to the nearest degree, of the coffee after 15 minutes, 30 min and 40 min (rounf to one decimal place). round
b) How much time did it take for the coffee to reach an internal temperature of 31°C?
c) What is the lowest possible temperature the coffee can reach?

a) 15 min $\Rightarrow T(15) = 80(0.5)^{\frac{15}{15}} + 18 = 58°C$ | 30 min $\Rightarrow T(30) = 80(0.5)^{\frac{30}{15}} + 18 = 38°C$ | 40 min: $T(40) = 80(0.5)^{40/15} + 18 = 30.6°C$

b) We want "t" when $T(t) = 31°C$

goal: get the exponential part "$(0.5)^{t/15}$" by itself.

$\Rightarrow 31 = 80(0.5)^{t/15} + 18$ $\quad (-18)$

$\Rightarrow 13 = 80(0.5)^{t/15}$ $\quad (\div 80)$

$\Rightarrow \frac{13}{80} = 0.5^{t/15}$

now that the exponential is isolated we can invert it by using a logarithm

$\frac{13}{80} = 0.1625$ ← if you prefer decimals

Now we take the log of both sides.

$\Rightarrow \log\left(\frac{13}{80}\right) = \left(\frac{t}{15}\right) \cdot \log(0.5)$ $\quad (\div \log(0.5))$

The "log rule" we are using is this: $\log(b^x) = x \cdot \log(b)$ — the exponent "comes down front".

$\Rightarrow \frac{\log(0.1625)}{\log(0.5)} = \frac{t}{15}$ $\quad (\times 15)$

$\Rightarrow t = \frac{(15)(\log(0.1625))}{\log(0.5)} = 39.3 \text{ min}$

c) The lowest possible temperature is given by the horizontal asymptote: $y = 18$

$\therefore 18°C.$

6. The population of a town has grown at an annual rate of approximately 2.2%. growth $\Rightarrow P(t) = P_0(1+r)^t$ $r = 0.022$ convert from a %!
Write an exponential function which will describe the town's population over time.
a) What is the town's population after 10 years have passed?
b) How long will it take for its population of 38 125 people to double at this growth rate? Explain how you found your answer.

$1+r$!

a) $P(t) = P_0(1.022)^t$ $P_0 = 38\,125$ given in part b $\Rightarrow$ shame on me

$\Rightarrow P(10) = 38\,125(1.022)^{10} = 47\,393$ people

b) We want 't' when $P(t) = 2(38\,125) = 76\,250$

$\Rightarrow 76250 = 38125(1.022)^t$ ← we need to use 'logs'

$\div 38125$ $\div 38125$

$\Rightarrow 2 = (1.022)^t$

$\Rightarrow \log(2) = t \cdot \log(1.022)$

$\div \log(1.022)$ $\div \log(1.022)$

$\Rightarrow t = \dfrac{\log(2)}{\log(1.022)} = 31.85$ $\Rightarrow$ Almost 32 years for the population to double

7. In 2001, a sum of \$4000 is invested at a rate of 6.5% per year for 5 years. What is the value of the investment when it matures? after 5 years

This is a "growth" problem $\Rightarrow A(t) = A_0(1+r)^t$ $r = 0.065$, $t = 5$, $A_0 = \$4000$ ← original investment

$\Rightarrow A(5) = 4000(1.065)^5 = \$5480.35$

$\therefore$ The investment is worth \$5480.35

#8 on next page

9. The half time for a certain type of radiation is 4.5 weeks. The amount of radioactive material after t weeks is

described by $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{4.5}}$ where A_0 is the initial mass.

~~Determine the mass of radioactive matrerial after 18 weeks~~ if there was 250g or radioactive material initially. $- A_0$

Q. for thought - will there ever be zero radiactive material? Explain.

$A(18) = 250\left(\frac{1}{2}\right)^{18/4.5}$

$= 15.625$ grams

Technically: No: the horizontal asymptote "$y=0$" means 0 grams of radioactive material

But - the function never hits the the horizontal asymptote.

8. Draw (basic) sketches of the following. Indicate the horizontanl asymptote, two points (use $x = -1, 1$ as domain values for those two points) and the y_{int}. $\Rightarrow x = 0$

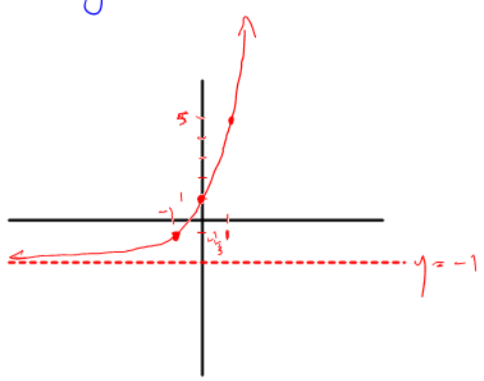
a) $f(x) = 2 \cdot (3)^x - 1$ growth!

b) $g(x) = \left(\frac{1}{2}\right)^x + 3$ decay!

a): points

x	$f(x) = 2 \cdot 3^x - 1$
-1	$2 \cdot 3^{-1} - 1 = -\frac{1}{3} \Rightarrow (-1, -\frac{1}{3})$
0	$2 \cdot 3^0 - 1 = 1 \Rightarrow (0,1)$
1	$2 \cdot 3^1 - 1 = 5 \Rightarrow (1,5)$

H.A.: $y = -1$



points

x	$g(x) = (\frac{1}{2})^x + 3$
-1	$(\frac{1}{2})^{-1} + 3 = 5 \Rightarrow (-1, 5)$
0	$(\frac{1}{2})^0 + 3 = 4 \Rightarrow (0, 4)$
1	$(\frac{1}{2})^1 + 3 = 3.5 \Rightarrow (1, 3.5)$

HA $y = 3$