11U – Trigonometric Identities Mini-Assignment   Name Sol'ns

Please hand in solutions to the following Trig Identities

Prove each Identity

1. $\frac{1}{\cos^2(x)} + \frac{1}{\sin^2(x)} = \csc^2(x) \cdot \sec^2(x)$

$LHS = \frac{1}{\cos^2(x)} + \frac{1}{\sin^2(x)}$ Comm. denom $(\cos^2(x))(\sin^2(x))$

$= \frac{\sin^2(x)}{\sin^2(x) \cdot \cos^2(x)} + \frac{\cos^2(x)}{\sin^2(x)\cos^2(x)}$

ring-a-ding-ding.

$= \frac{\sin^2(x) + \cos^2(x)}{\sin^2(x) \cdot \cos^2(x)}$

$= \frac{1}{\sin^2(x) \cdot \cos^2(x)} = \frac{1}{\sin^2(x)} \cdot \frac{1}{\cos^2(x)} = \csc^2(x) \cdot \sec^2(x)$

$= RHS$ □

2. $\sin(x) \cdot \tan(x) = \frac{1-\cos^2(x)}{\cos(x)}$

$LHS = \sin(x) \cdot \tan(x)$

$= \sin(x) \cdot \frac{\sin(x)}{\cos(x)}$

by pythagorean trig. identity.

$= \frac{\sin^2(x)}{\cos(x)} = \frac{1-\cos^2(x)}{\cos(x)}$

$= RHS$ □.

3. $(1 + \tan^2(x))(1 - \cos^2(x)) = \tan^2(x)$

$LHS = (1 + \tan^2(x))(1 - \cos^2(x))$

what we want → leave alone!

$= \underbrace{1 - \cos^2(x)}_{\sin^2(x)} + \boxed{\tan^2(x)} - \tan^2(x) \cdot \cos^2(x)$

$= \sin^2(x) + \tan^2(x) - \frac{\sin^2(x)}{\cancel{\cos^2(x)}} \cdot \cancel{\cos^2(x)}$

$= \sin^2(x) + \tan^2(x) - \sin^2(x)$

$= \tan^2(x)$

$= RHS$ □

4. $\tan(\alpha) + \frac{1}{\tan(\alpha)} = \csc(\alpha) \cdot \sec(\alpha)$

$LHS = \frac{\tan(\alpha)}{1} + \frac{1}{\tan(\alpha)}$ → Comm. denom. $\tan(\alpha)$

$= \frac{\tan(\alpha) \cdot \tan(\alpha)}{\tan(\alpha)} + \frac{1}{\tan(\alpha)}$

$= \frac{\tan^2(\alpha) + 1}{\tan(\alpha)}$

$= \frac{\sec^2(\alpha)}{\tan(\alpha)}$

$= \frac{\sec^2(\alpha)}{\frac{\sin(\alpha)}{\cos(\alpha)}}$

$= \sec^2\alpha \cdot \frac{\cos(\alpha)}{\sin(\alpha)}$

$= \sec^{\cancel{2}\,1}(\alpha) \cdot \frac{\csc(\alpha)}{\cancel{\sec(\alpha)}}$

$= \sec(\alpha) \cdot \csc(\alpha)$

$= RHS$ □

5. $\cos^4(\phi) - \sin^4(\phi) = 1 - 2\sin^2(\phi)$

$$\text{LHS} = \cos^4(\phi) - \sin^4(\phi)$$

ring-a-ding-ding

$$= \left(\cos^2(\phi) - \sin^2(\phi)\right)\underbrace{\left(\cos^2(\phi) + \sin^2(\phi)\right)}_{1}$$

$$= \cos^2(\phi) - \sin^2(\phi)$$

$$\text{RHS} = \underset{R}{1} - 2\sin^2(\phi)$$

$$= \cos^2(\phi) + \sin^2(\phi) - 2\sin^2(\phi)$$

$$= \cos^2(\phi) - \sin^2(\phi)$$

$$= \text{LHS} \quad \square$$

6. $\dfrac{\tan(\theta)+\cos(\theta)}{\sin(\theta)} = \sec(\theta) + \cot(\theta)$

$$\text{LHS} = \frac{\tan(\theta) + \cos(\theta)}{\sin(\theta)}$$

$$= \frac{\tan(\theta)}{\sin(\theta)} + \frac{\cos(\theta)}{\sin(\theta)}$$

$\left(\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}\,!\right)$

because $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$

$$= \frac{\cancel{\sin(\theta)}}{\cos(\theta)} \cdot \frac{1}{\cancel{\sin(\theta)}} + \cot(\theta)$$

$$= \frac{1}{\cos(\theta)} + \cot(\theta)$$

$$= \sec(\theta) + \cot(\theta)$$

$$= \text{RHS} \quad \square$$