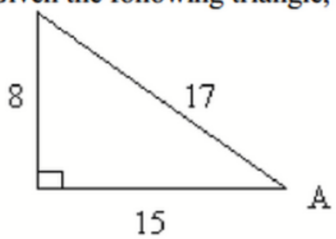
14. Given the following triangle, state the six trigonometric ratios for $\angle A$.

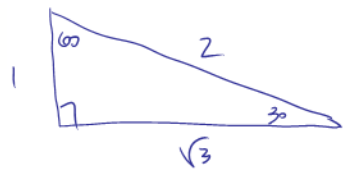


$\sin A = \frac{8}{17}$ $\quad \csc A = \frac{17}{8}$

$\cos A = \frac{15}{17}$ $\quad \sec A = \frac{17}{15}$

$\tan A = \frac{8}{15}$ $\quad \cot A = \frac{15}{8}$

15. Determine the exact value of $2\sin^2 60° \times \tan 30°$.



$2\sin^2(60) \times \tan(30)$

$= 2\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{\sqrt{3}}\right)$

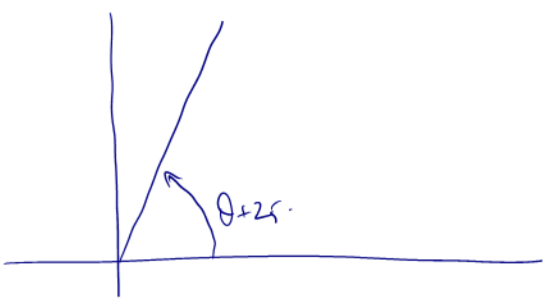
acceptable to me

rationalizing the denominator

required by some people

$= 2\left(\frac{3}{4}\right)\left(\frac{1}{\sqrt{3}}\right) = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{2(3)} = \frac{\sqrt{3}}{2}.$

16. Given $\cos(\theta + 25°) = 0.2237$, solve for θ to the nearest degree. Assume θ is in quadrant 1.



$\cos(\theta + 25°) = 0.2237$

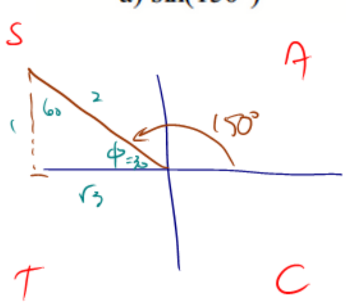
$\Rightarrow \theta + 25° = \cos^{-1}(0.2237)$

$= 77°$

$\Rightarrow \theta = 77 - 25 = 52°.$

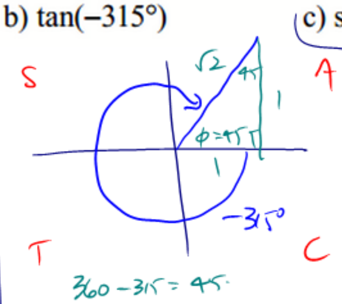
17. For each question, draw the angle of roation and determine the EXACT trig ratio:

a) $\sin(150°)$



$\sin(150°) = +\frac{1}{2}$

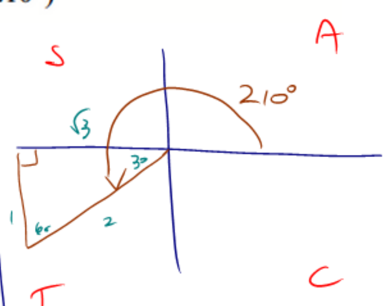
b) $\tan(-315°)$



$360 - 315 = 45.$

$\tan(-315°) = +1$

c) $\sec(210°)$



$\sec(210°) = \frac{hyp}{adj}$

$\frac{1}{\cos(210°)}$

$\sec(210) = -\frac{2}{\sqrt{3}} \left(= -\frac{2\sqrt{3}}{3}.\right)$

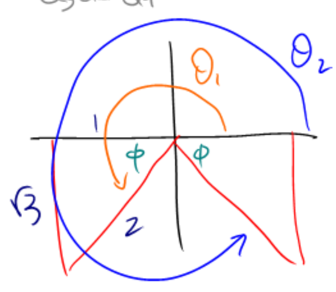
acceptable.

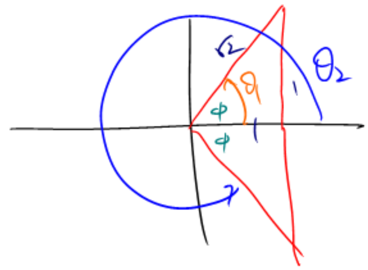
18. Given the trig ratio determine **both possible** values for θ where $0° \le \theta \le 360°$:

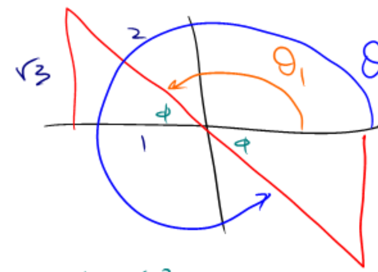
Q2 or Q4

a) $\sin(\theta) = -\frac{\sqrt{3}}{2}$ b) $\cos(\theta) = +\frac{1}{\sqrt{2}}$ Q1 or Q4 c) $\tan(\theta) = -\sqrt{3}$

Q3 or Q4







$\phi = 60°$ $\phi = 45°$ $\phi = 60°$

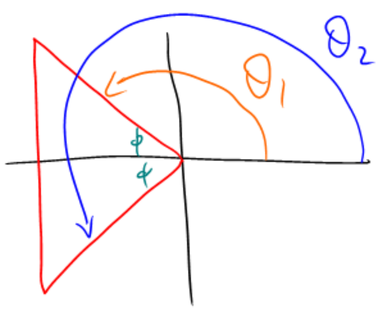
$\theta_1 = 180 + \phi = 240°$ $\theta_1 = 45°$ $\theta_1 = 180 - \phi = 120°$

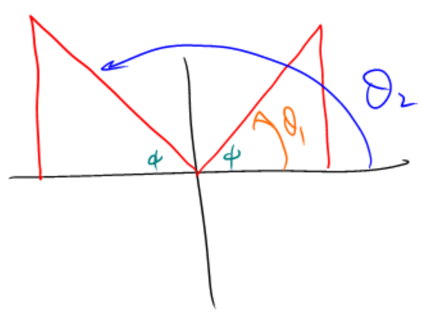
$\theta_2 = 360 - \phi = 300°$ $\theta_2 = 360 - \phi = 315°$ $\theta_2 = 360 - \phi = 300°$

19. Given the (inexact!!) trig ratio, determine **both possible** values for θ where $0° \le \theta \le 360°$ (round to the nearest degree):

tells us Q2 & Q3 reciprocal of SINE tells us Q1 & Q2

a) $\cos(\theta) = -0.3421$ b) $\csc(\theta) = +1.512$





IGNORE THE NEGATIVE and find ϕ!

$\cos(\phi) = +0.3421$

$\Rightarrow \phi = \cos^{-1}(0.3421)$

$= 71°$

$\theta_1 = 180 - \phi = 109°$

$\theta_2 = 180 + \phi = 251°$

$\csc(\phi) = 1.512$

$\Rightarrow \sin(\phi) = \frac{1}{1.512} = 0.661$

$\Rightarrow \phi = \sin^{-1}(0.661) = 41°$

$\theta_1 = 41°$

$\theta_2 = 180 - \phi = 139°$

20. Prove the following identity.

$$1 = \frac{(\sin^4 x - \cos^4 x)}{\tan x \sin x \cos x - \cos^2 x}$$

$$RHS = \frac{\sin^4 x - \cos^4 x}{\tan x \cdot \sin x \cdot \cos x - \cos^2 x}$$

$$= \frac{(\sin^2 x - \cos^2 x)\underbrace{\cancel{(\sin^2 x + \cos^2 x)}}_{\to 1}}{\frac{\sin x}{\cancel{\cos x}} \cdot \sin(x) \cdot \cancel{\cos(x)} - \cos^2 x}$$

$\sin^2 x$!

$$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x - \cos^2 x} = 1 = LHS \quad \square$$

21. Prove the following identity.

$$\tan x \sec x = \frac{\sin^3 x + \cos^2 x \sin x}{\cos^2 x}$$

$$RHS = \frac{\sin^3 x + \cos^2 x \cdot \sin x}{\cos^2 x}$$

$$= \frac{\sin(x)\underbrace{\cancel{(\sin^2 x + \cos^2 x)}}_{\to 1}}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

(you might see the solution here: $\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \cdot \sec x$ Boom)

OR $\boxed{\text{switch sides}}$

switching sides

$$LHS = \tan(x) \cdot \sec(x)$$

$$= \frac{\sin x}{\cos(x)} \cdot \frac{1}{\cos(x)}$$

$$= \frac{\sin x}{\cos^2 x} = RHS \quad \square$$

22. A triangular plot of land is enclosed by a fence. One side of the fence is 8.1 m long with an opposite angle of 75°. An adjacent side of the fence is 5.7 m long with an opposite angle of θ.
a) Make a sketch of the situation.
b) Determine θ to the nearest degree.

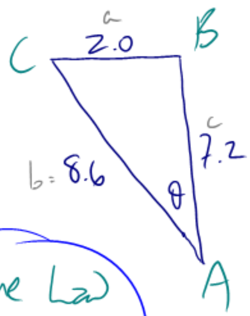
a)

b) $$\frac{\sin\theta}{5.7} = \frac{\sin 75}{8.1}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{(5.7)(\sin(75))}{8.1}\right)$$

$$\doteq 43°$$

23. The posts of a hockey goal are 2.0 m apart. A player attempts to score by shooting the puck along the ice from a point 7.2 m from one post and 8.6 m from the other. Within what angle θ must the shot be made? Round your answer to the nearest degree.



Cosine Law for angles

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

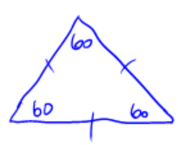
$$\Rightarrow \cos A = \frac{8.6^2 + 7.2^2 - 2.0^2}{2(8.6)(7.2)}$$

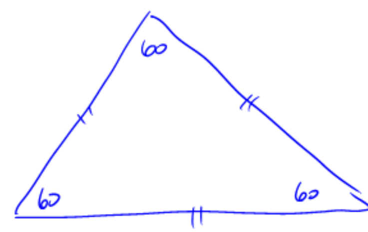
$$\Rightarrow A = \cos^{-1}\left(\frac{8.6^2 + 7.2^2 - 2.0^2}{2(8.6)(7.2)}\right) \doteq 10^\circ$$

$\therefore$ The player needs to shoot within an angle of 10°.

24. Given a triangle with 3 angles that sum to 180°, can the lengths of the sides be determined?

No - we need at least one side to determine the $\triangle$

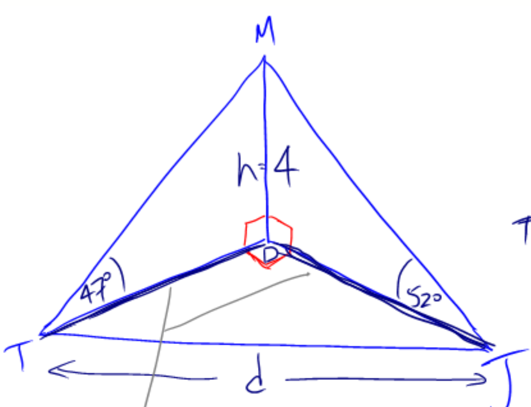




⟸ Different sized $\triangle$s with equal angles.

No way to calculate the sides.

25. Mary stands on a balcony. Joe is on the left of the balcony looking up at her at an angle of 52° with the ground. Trent is on the right of the balcony looking up at her at an angle of 47° with the ground. If the height, *h*, is 4 m, how far apart are Joe and Trent standing to the nearest tenth of a metre? Assume the angle the base of the balcony makes between Joe and Trent is 90°.



both of these sides are "common" sides

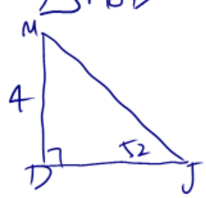
$\triangle MTD$

$\tan(47) = \dfrac{4}{TB}$

$\Rightarrow TB = \dfrac{4}{\tan(47)}$

$= 3.73$

$\triangle MJD$

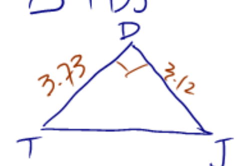


$\tan(52) = \dfrac{4}{DJ}$

$\Rightarrow DJ = \dfrac{4}{\tan(52)}$

$\doteq 3.12$

$\triangle TDJ$



By pythagorus:

$TJ^2 = 3.73^2 + 3.12^2$

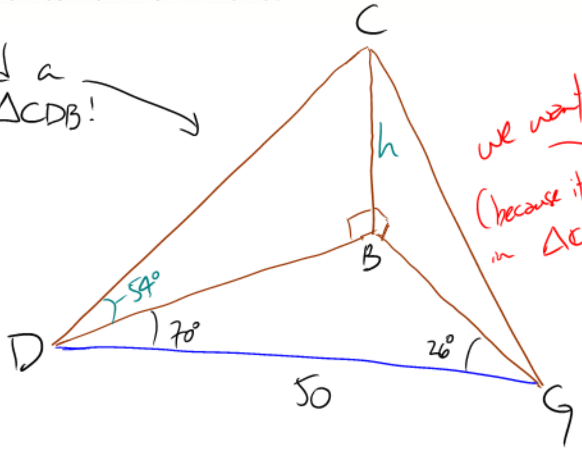
$\Rightarrow TJ = \sqrt{\quad ''\quad}$

$= 4.9$ m

$\therefore$ Trent and Joe are 4.9 m apart.

26. Doug is looking at a cliff. He determines that the angle of elevation to the top is 54° from where he is at. 50 m away from Doug, Gary estimates the angle between the base of the cliff, himself, and Doug to be 26° while Doug estimates the angle between the base of the cliff, himself, and Gary to be 70°. What is the height, *h*, of the cliff to the nearest tenth of a metre?

need a side in $\triangle CDB$!

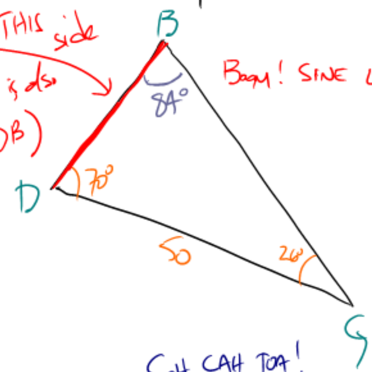


$\triangle DBG$

$\angle B = 180 - 70 - 26 = 84°$

we want THIS side (because it is also in $\triangle CDB$)

Bam! SINE LAW



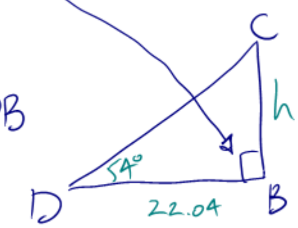
$$\frac{DB}{\sin(26)} = \frac{50}{\sin 84}$$

$$\Rightarrow DB = \frac{50 \cdot \sin(26)}{\sin(84)}$$

$$= 22.04 \text{ m.}$$

SOH CAH TOA!

$\triangle CDB$



$$\tan(54) = \frac{h}{22.04}$$

$$\Rightarrow h = (22.04)(\tan(54))$$

$$= 30.3 \text{ m}$$

$\therefore$ The cliff is approximately 30.3 m tall.

I hope this has helped!