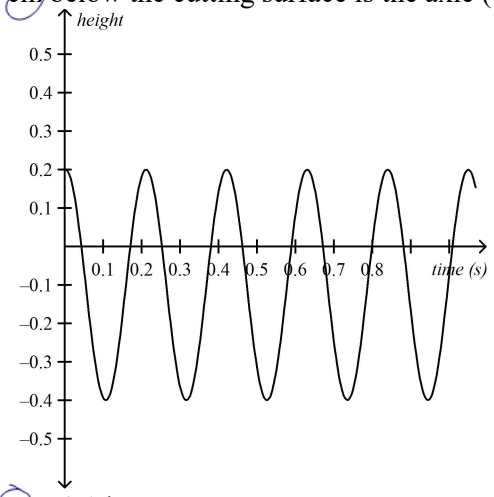
11U5: Sinusoidal Functions - Practice Test W24

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. The following function represents the height of a saw tooth in inches after time in seconds for $t > 0$. How many cm below the cutting surface is the axle (the center) of the blade?



CA: $y = \frac{max + min}{2}$

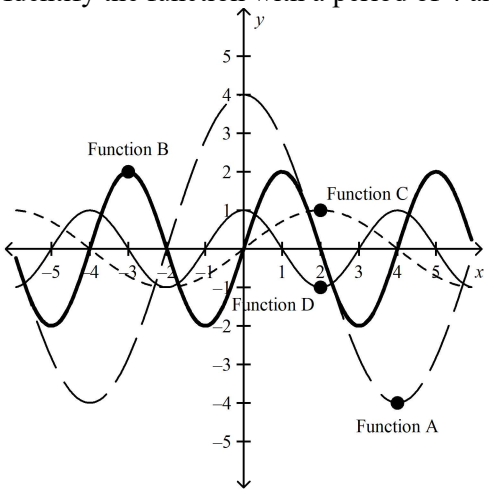
$= \frac{0.2 + (-0.4)}{2}$

$= \frac{-0.2}{2}$

$= -0.1$ → 0.1 in below

a. 0.1 in.
b. 0.2 in.
c. 0.3 in.
d. 0.4 in.

2. Identify the function with a period of 4 and an amplitude of 2.



A: $P = 8$

B: $P = 4$ $a = 2$

C: $P = 8$

D $P = 4$ $a = 1$

a. Function A
b. Function B
c. Function C
d. Function D

3. Which of the following situations would produce a periodic graph?
   a. passing a basketball, but the ball misses the target and just bounces along the ground
   b. a machine that cuts a board every 10 s
   c. a biker's pace through a mountainous terrain
   d. jumping as high as you can for 10 straight minutes

4. Determine the period of the function $y = \cos(2x) - 1$.

$P = \frac{360}{k} = \frac{360}{2} = 180°$

B

a. 90° c. 270°

b. 180° d. 360°

5. A spring bounces up and down according to the model $d(t) = 4\sin(30t) + 3$, where $d(t)$ is the displacement in cm from the rest position and t is time in seconds. What is the equation of the central axis?

B

a. $y = 2$ c. $y = 4$

b. $y = 3$ d. $y = 5$

$y = 3$

Full Solution

Provide clear solutions to the following problems. Communications points are awarded for clarity in your presentation of the mathematics invovled in your solution.

6. Sketch two full cycles of the function $f(x) = -2\cos(3x - 180) + 2$.

"min - ca - max - ca - min"

standard form: $f(x) = -2\cos(3(x - 60)) + 2$

$a = 2$

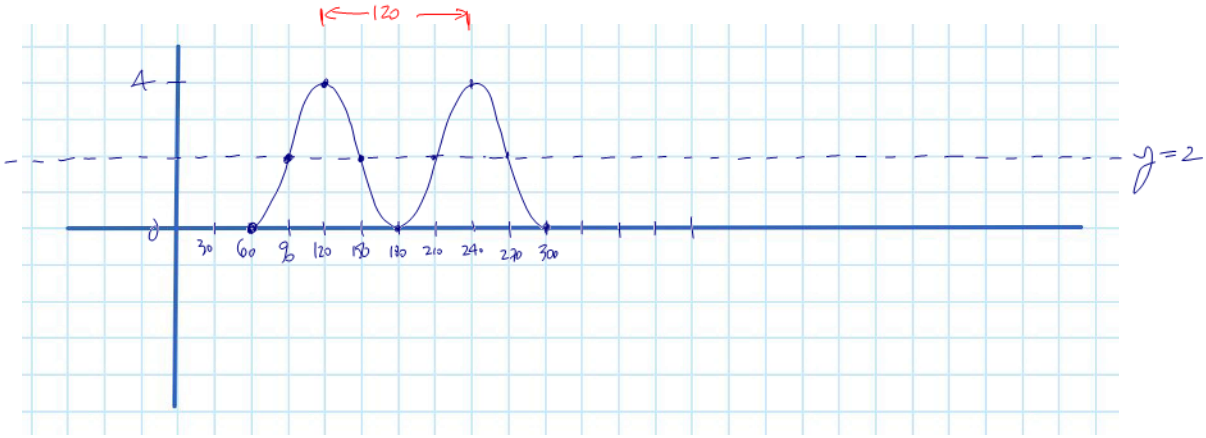
$P = \frac{360}{3} = 120$

$dx = \frac{120}{4} = 30$ (1 box)

P.S. 60 right (2 boxes)

CA: $y = 2$

scale: gcf $\{30, 60\} = 30$



7. Suppose the following data represents the height of an object after time t seconds. Determine the equation of the function that models the height of this object.

period = 120

t	0	30	60	90	120	150	180
height	15	8.5	2	8.5	15	8.5	2
	max	ca	min	ca	max		

+ cosine ✓ $d = 0$

$a = \frac{\max - \min}{2} = \frac{15 - 2}{2}$

$a = 6.5$

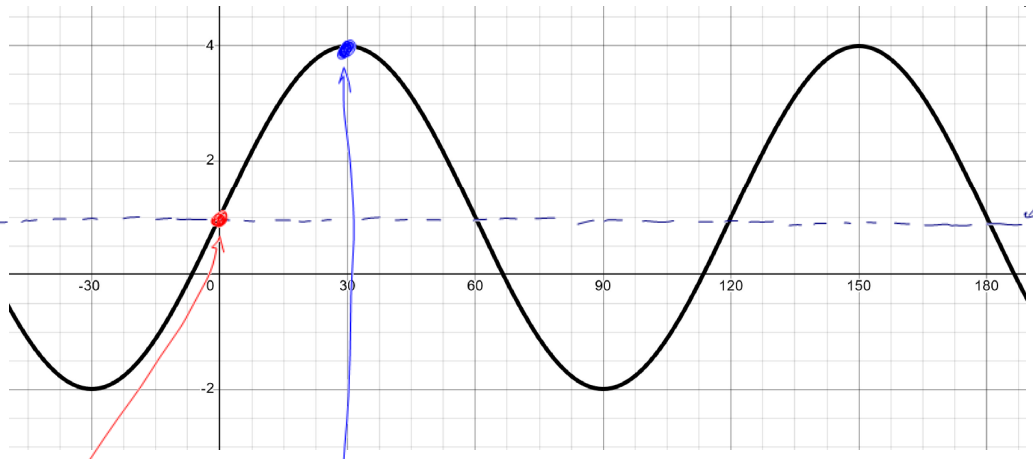
CA: $y = 8.5$

$k = \frac{360}{P} = \frac{360}{120}$

$k = 3$

$h(t) = 6.5\cos(3t) + 8.5$

8. Determine both a sine and a cosine function which represents the given sketch.



max = 4

min = −2

$a = \frac{max - min}{2} = \frac{4-(-2)}{2} = 3$

$y = 1$

$C: y = \frac{max + min}{2} = \frac{4+(-2)}{2}$

$y = 1$

$P = 120 \Rightarrow k = \frac{360}{120} = 3$

with $d = 0 \Rightarrow$ positive sine

with $d = 30$ right $\Rightarrow$ positive cosine.

Sine: $f(x) = 3\sin(3x) + 1$

Cosine: $g(x) = 3\cos(3(x-30)) + 1$

9. Fred is riding a Ferris wheel while his friends track his motion using a stop watch. The friends notice that Fred reaches the maximum height of 18*m* at 10 sec, and then reaches a minimum height of 2*m* at 40 sec. Determine a sinusoidal equation which describes Fred's motion around the wheel.

phase shift of 10 $\Rightarrow$ positive cosine.

max (10, 18) $d = 10$

min (40, 2)

max − to − min is half a period

$\Rightarrow \frac{1}{2}P = 30$ seconds (40 − 10)

$\Rightarrow P = 60$

$\therefore k = \frac{360}{P} = \frac{360}{60}$

$k = 6$

$a = \frac{max - min}{2} = \frac{18-2}{2}$

$a = 8$

$C = \frac{max + min}{2} = \frac{18+2}{2}$

$C = 10$

$\therefore$ eqn $h(t) = 8\cos(6(t-10)) + 10$

10. Lydia holds the end of a yo-yo string 0.9 metres from the ground and swings the yo-yo in a circle perpendicular to the ground. After 0.5 seconds, the yo-yo is at it's closest point to the ground, 0.1 metres. After 1 second, the yo-yo is at its farthest point from the ground, 1.7 metres.
a) What is the amplitude of the function that represents the yo-yo's distance from the ground in terms of the seconds that have passed, and what does it represent?
b) What is the period of the function representing the spinning yo-yo and what does the period represent?

min (0.5, 0.1)

max (1, 1.7)

a) $a = \frac{max - min}{2} = \frac{1.7 - 0.1}{2} = 0.8$ m

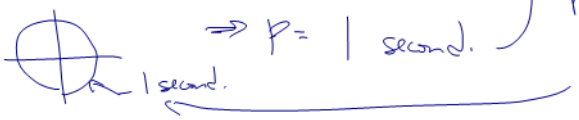
This is the length of the string.

b) min − to − max is $\frac{1}{2}$ period

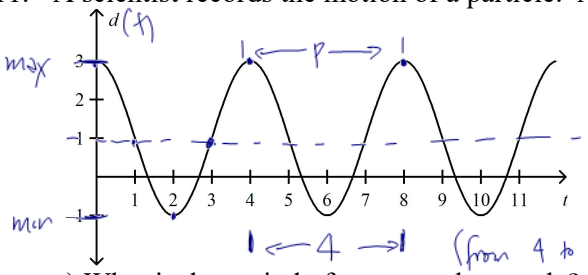
$\Rightarrow \frac{1}{2}P = 0.5$ seconds

$\Rightarrow P = 1$ second.

This means it takes 1 second for one full rotation of the yo-yo.



11. A scientist records the motion of a particle. A graph of that motion is shown below.



a) What is the period of one complete cycle?

b) What is the range of this function?

c) If the particle can survive for twenty complete cycles before deteriorating, determine the domain of the function.

a) 1 period is 4 time units (probably 4 seconds)

c) 1 period = 4 seconds

⇒ 20 periods ⇒ 80 seconds

$D: \{t \in \mathbb{R} \mid 0 \le t \le 80\}$

b) range: $\{d \in \mathbb{R} \mid \min \le d(t) \le \max\} = \{d(t) \in \mathbb{R} \mid -1 \le d(t) \le 3\}$

"y" - values.

12. Determine if the following tables might represent periodic functions. Explain your reasoning for each table.

a)

x	y
−2	1
−1	3
0	4
1	1
2	3
3	4
4	1

yes - the 'y-values' repeat at regular x-intervals

b)

x	1	2	4	8	16	32	64
y	3	10	3	10	3	10	3

1 2 4 8 16 32

No - the y-values repeat, but not at regular x-intervals

c)

x	y
−12	12
−7	10
−2	15
3	18
8	14
13	9
18	22

No: the y-values don't repeat at all.