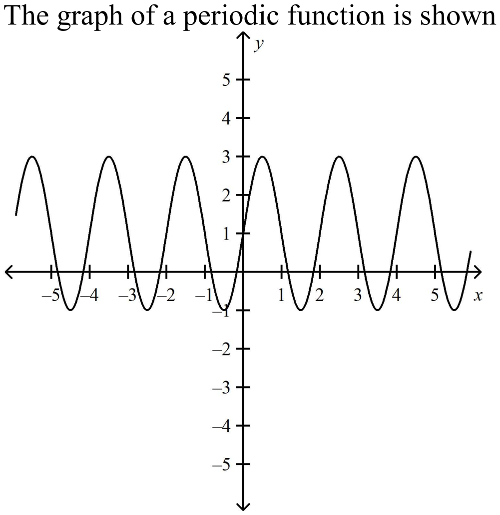
11U6 F23 - Sinusoidal Functions Practice

Multiple Choice
Identify the choice that best completes the statement or answers the question. Write the letter of your choice in the appropriate space on the Answer Sheet.

1. The graph of a periodic function is shown below. What is the equation of [the central] axis of the function?



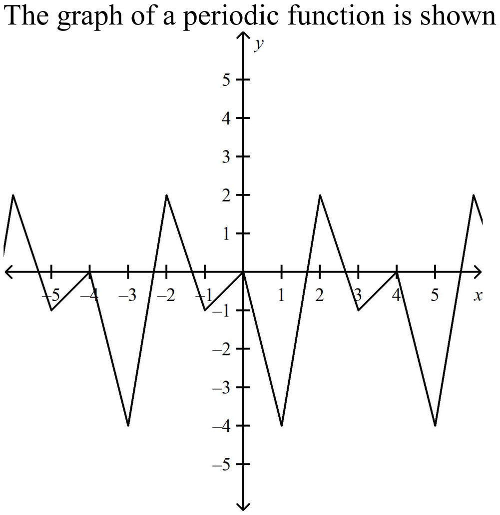
max = 3
min = −1

= (max + min)/2 = (3 + (−1))/2 = 2/2 = 1

a. $y = -1$ c. $y = 1$
b. $y = 0$ d. $y = 3$

C

2. The graph of a periodic function is shown below. What is the amplitude of the function?



max = 2
min = −4

a = (max − min)/2 = (2 − (−4))/2 = 6/2 = 3

a. 2 c. 4
b. 3 d. 6

B

3. Determine the amplitude of the function $y = 4\sin x - 7$ by ~~graphing the function~~.

a. 7 c. 4
b. −7 d. −4

C

4. Determine the equation of a sine function that would have a range of $\{y \in \mathbf{R} | 6 \leq y \leq 9\}$ and a period of 60°.
   a. $y = 3\sin(12x) + 6$
   b. $y = 3\sin(12x) + 9$
   c. $y = 1.5\sin(6x) + 7.5$
   d. $y = \sin(6x) + 9$

Sinusoidal Functions Practice Problems

5. Sketch two cycles of $f(x) = -3\cos(3x + 90^\circ) - 1$

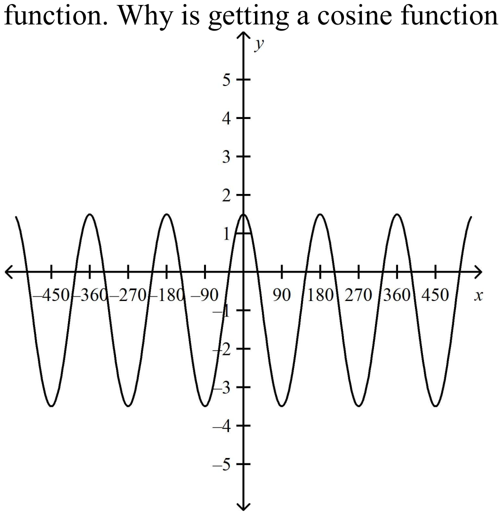
6. Suppose Ken is riding a Ferris wheel. The maximum height he attains is 16 m and the minimum height is 0 m. The time it takes to make one cycle is about 6 minutes. Draw a graph which represents this scenario if Ken starts at a height of about 9 m.

7. A dolphin is swimming in the ocean and jumping out at constant intervals. If it jumps 3 m high and then dives 2 m before jumping again, what is the amplitude of the function that can be modelled by this situation?

8. Determine the range of the function $K(t) = 6\cos(90t) - 10$.

9. Without graphing, determine the amplitude, period, domain, and range of the function $y = 1.9\cos(12x) - 11$.

10. Determine two equations for the sinusoidal below. One equation needs to be a cosine function and the other a sine function. Why is getting a cosine function "better" than a sine function in this case?



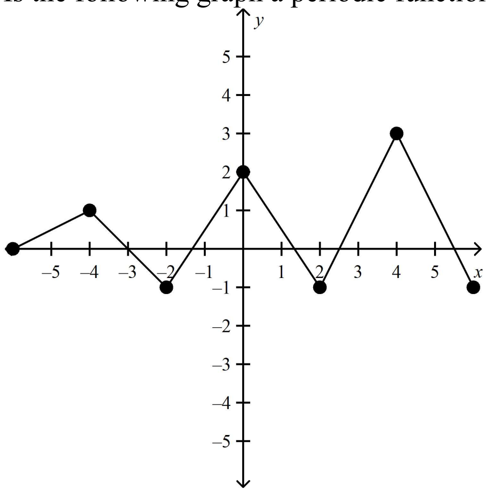
11. For the following table of data, determine an equation of a sine function that satisfies the given data.

x	−60°	−30°	0°	30°	60°	90°	120°
y	10	8	6	8	10	8	6

12. A sinusoidal function has an amplitude of 6 units, a period of 45°, and a minimum at (0, 3). Determine an equation of the function. (A simple sketch may be helpful but is not necessary.) Give the coordinates of the next minimum (moving to the right from the given minimum (0, 3))

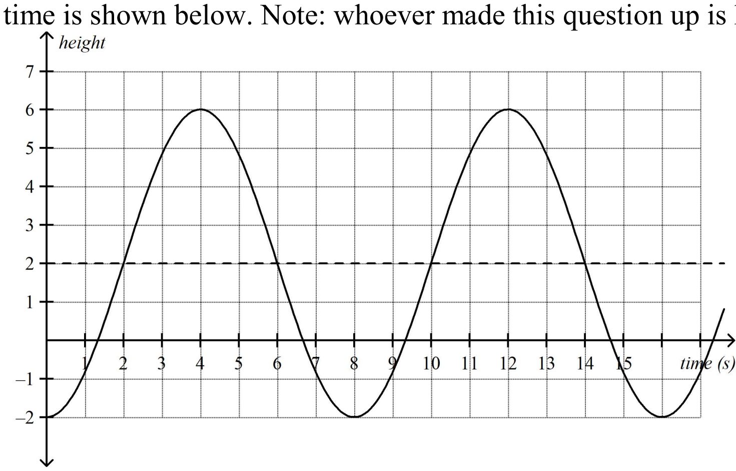
13. The top of a flagpole is swaying in the wind. The top sways from 6 cm to the right of its resting position (+6 cm) to 6 cm to the left of its resting position (–6 cm) and back to the right 6 times per second. Determine the equation of a sinusoidal function that describes the distance the top of the pole is from its resting position in terms of time.

14. Is the following graph a periodic function? Explain why or why not.



15. A man delivers packages from location A to location B and then returns to location A to get more packages. He does this in equal amounts of time each trip, but takes a 10 minute break when he gets to location B. Draw a possible sketch of this situation. Is it periodic? Explain.

16. A fish is caught in a water wheel. A graph of its height (in metres) with respect to the surface of the water over time is shown below. Note: whoever made this question up is kind of mean. It wasn't Mr. Templeton!



a) What is the diameter of the water wheel?
b) How long does it take the fish to complete one revolution of the wheel?
c) Does the fish reenter the water? If so, for approximately how long?
d) After 36 seconds, at what height will the fish be?
e) Give an equation for the curve as a sine function.

17. Using a graphing calculator, graph each of the following functions. Determine if the functions are periodic. If so, determine if they are sinusoidal (sinusoidal means having the basic shape of a sine or cosine function).
a) $y = x\sin x$
b) $y = \sin x - \cos x$

18. The size of a certain population of wild horses varies because of certain ecological pressures. In 1999, the number of horses was 100. In 2002, the population was down to 60, but in 2005, the population was back up to 100.
a) Assume that 100 was the maximum number of horses and 60 was the minimum. Write an equation that models the size of the population of horses, in terms of what year it is, starting with 1999.
b) What is the equation of the axis of this equation, and what does it represent?
c) How many horses were there in 2000?
d) How many horses should there be in 2015?

Sol'ns

5. Sketch two cycles of $f(x) = -3\cos(3x + 90^\circ) - 1$

STANDARD FORM $f(x) = a\cos(k(x-d)) + c$

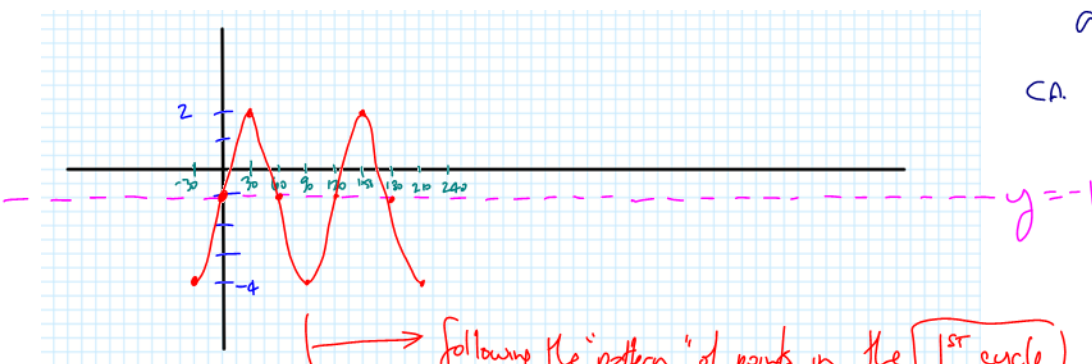
$\rightarrow f(x) = -3\cos(3(x+30)) - 1$

$a = -3$

$k = 3 \Rightarrow P = \frac{360}{3} = 120$

C.A. $y = -1$

$d = 30$ left



following the "pattern" of points in the 1st cycle → this is sketched from these points

ToV.

parent: $g(x) = \cos(x)$

Transformed $f(x) = -3\cos(3(x+30)) - 1$

use your calculator if needed

x_p	g
0	1
90	0
180	-1
270	0
360	1

the usual "axis angles"

$x_T = \frac{1}{3}x_p - 30$	$f = -3g - 1$
-30	-4
0	-1
30	2
60	-1
90	-4

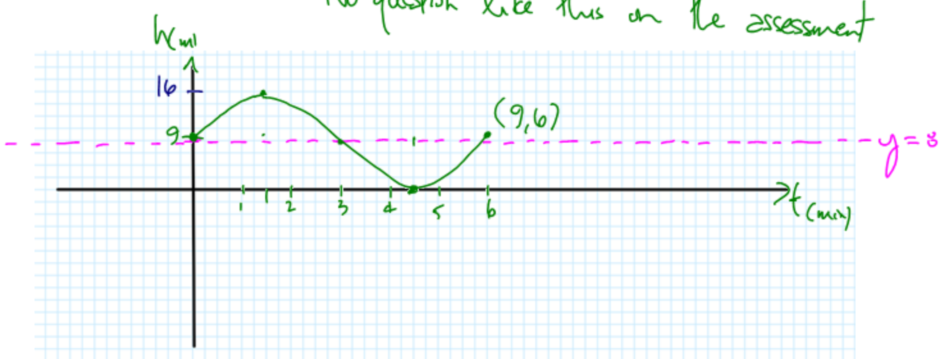
plot these points

⇒ follow the "pattern" after these point

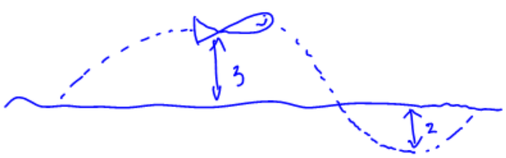
6. Suppose Ken is riding a Ferris wheel. The maximum height he attains is 16 m (max) and the minimum height is 0 m (min). The time it takes to make one cycle is about 6 minutes. Draw a graph which represents this scenario if Ken starts at a height of about 9 m.

No question like this on the assessment

$a = \frac{max - min}{2} = \frac{16 - 0}{2} = 8$ $\qquad$ $CA = \frac{max + min}{2} = \frac{16 + 0}{2} = 8$



7. A dolphin is swimming in the ocean and jumping out at constant intervals. If it jumps 3 m high (max (+3)) and then dives 2 m (min (−2)) before jumping again, what is the amplitude of the function that can be modelled by this situation?



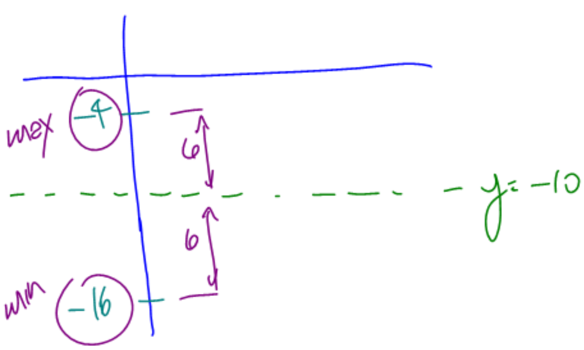
$a = \frac{max - min}{2} = \frac{3 - (-2)}{2} = \frac{5}{2} = 2.5\,m$

8. Determine the range of the function $K(t) = 6\cos(90t) - 10$. $\qquad a = 6, \; CA = -10$

"Range" is the amount of the vertical axis (y-axis) that is involved with the fn. That is, the range looks like this:

need these two #s

$R: \{K(t) \in \mathbb{R} \mid min \le K(t) \le max\}$



max is CA + amplitude = −10 + 6 = −4

min is CA − amplitude = −10 − 6 = −16

$\therefore R = \{K(t) \in \mathbb{R} \mid -16 \le K(t) \le -4\}$

9. Without graphing, determine the amplitude, period, domain, and range of the function $y = 1.9\cos(12x) - 11$.

(here you are expected to know all of the "parts" of a sinusoidal fn → a, k, d, c)

doesn't ask for a number of cycles

$a = 1.9$ $\qquad$ $P = \frac{360}{k} = \frac{360}{12} = 30°$

$CA: y = -11$ $\qquad$ $d = 0$

min = CA − amplitude = −11 − 1.9 = −12.9

max = CA + amplitude = −11 + 1.9 = −9.1

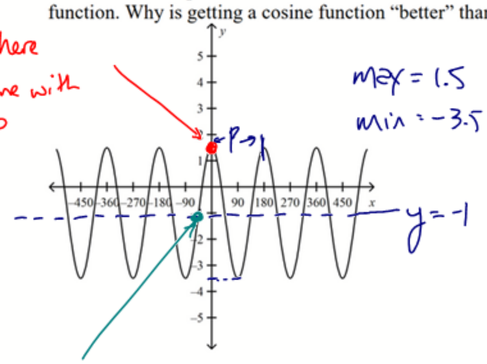
no restrictions because it doesn't

$D: \{x \in \mathbb{R}\}$ $\qquad$ $R = \{y \in \mathbb{R} \mid min \le y \le max\} = \{y \in \mathbb{R} \mid -12.9 \le y \le -9.1\}$

$y = a \,\substack{\sin \\ \cos}\,(k(x - d)) + c$

$y = a\,\sin/\cos(k(x-d)) + c$

10. Determine two equations for the sinusoidal below. One equation needs to be a cosine function and the other a sine function. Why is getting a cosine function "better" than a sine function in this case?



max = 1.5, min = -3.5

$a = \frac{max - min}{2} = \frac{1.5-(-3.5)}{2} = 2.5$

CA: $y = \frac{max+min}{2} = \frac{1.5+(-3.5)}{2} = -1$

$P = 180° \Rightarrow k = \frac{360}{P} = \frac{360}{180} = 2$

To write an equation (model) we need 5 things: amplitude, Central Axis, k, d and whether it's a sine or cosine. (we are doing both)

Cosine: $f(x) = 2.5\cos(2x) - 1$

Sine: $g(x) = 2.5\sin(2(x+45°)) - 1$

11. For the following table of data, determine an equation of a sine function that satisfies the given data.

x	−60°	−30°	0°	30°	60°	90°	120°
y	10	8	6	8	10	8	6
	max	CA	min	CA	max	CA	min

"start on central axis" — d = 30°

starting here means −sine. starting here means +ve sine. max-to-max is 1 period.

amplitude = 2 (max is 10, CA is 8)

CA: y = 8

$P = 120° \rightarrow k = \frac{360}{P} = \frac{360}{120} = 3$

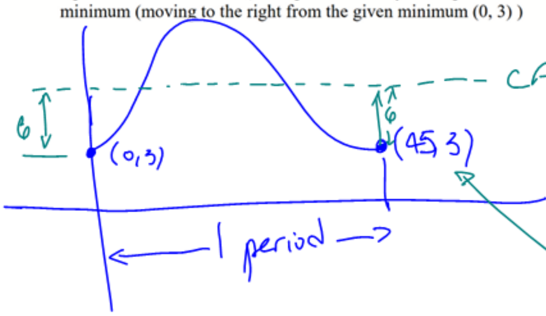
$f(x) = a\sin(k(x-d)) + c$

$\rightarrow f(x) = 2\sin(3(x-30)) + 8$

12. A sinusoidal function has an amplitude of 6 units, a period of 45°, and a minimum at (0, 3). Determine an equation of the function. (A simple sketch may be helpful but is not necessary.) Give the coordinates of the next minimum (moving to the right from the given minimum (0, 3))

neg. cosine; on the y-axis ⇒ d = 0; a = 6

$P = 45° \Rightarrow k = \frac{360}{45} = 8$



CA: y = 9 (min + amplitude)

$g(\theta) = -6\cos(8x) + 9$

the next min is 1 period (45°) to the right

$(0,3) \longrightarrow (0+45, 3) = (45,3)$

13. The top of a flagpole is swaying in the wind. The top sways from 6 cm to the right of its resting position (+6 cm) to 6 cm to the left of its resting position (–6 cm) and back to the right 6 times per second. Determine the equation of a sinusoidal function that describes the distance the top of the pole is from its resting position in terms of time.

$a = 6$

CA: $y = 0$

6 cycles/sec

$\Rightarrow \frac{1}{6}$ sec/cycle.

$\Rightarrow p = \frac{1}{6}$

$\Rightarrow k = \frac{360}{p} = \frac{360}{\frac{1}{6}}$

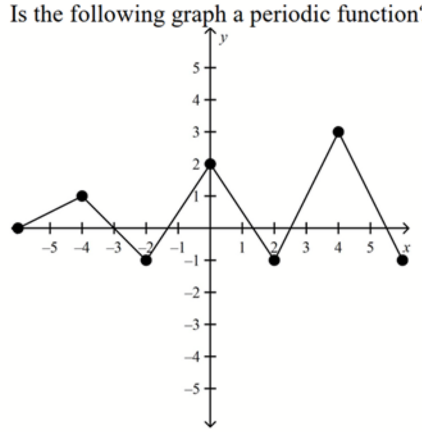
$k = 2160$

Assuming we begin with the pole at "rest position"

→ sine with $d = 0$

$$f(x) = 6\sin(2160x)$$

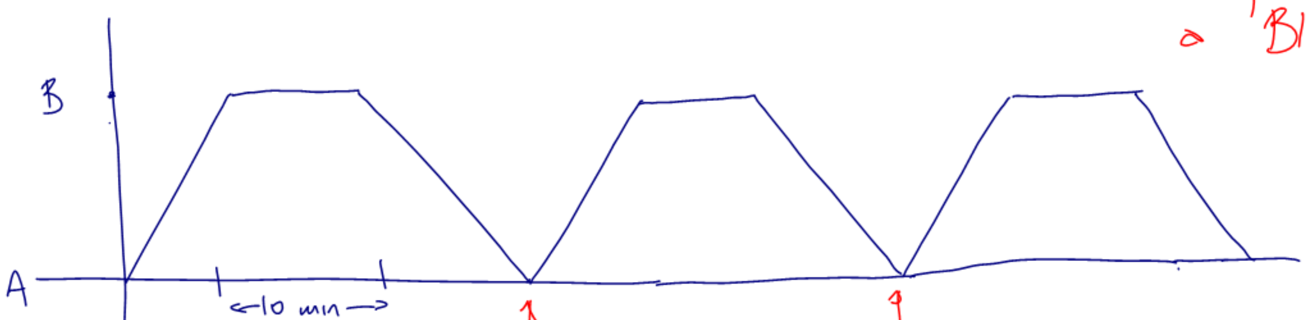
14. Is the following graph a periodic function? Explain why or why not.



This fn is NOT periodic because the y-values do not repeat from cycle to cycle

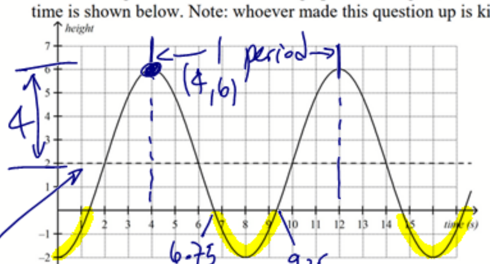
15. A man delivers packages from location A to location B and then returns to location A to get more packages. He does this in equal amounts of time each trip, but takes a 10 minute break when he gets to location B. Draw a possible sketch of this situation. Is it periodic? Explain.

This is periodic with a BIG assumption



no resting at location A

16. A fish is caught in a water wheel. A graph of its height (in metres) with respect to the surface of the water over time is shown below. Note: whoever made this question up is kind of mean. It wasn't Mr. Templeton!



a) What is the diameter of the water wheel?
b) How long does it take the fish to complete one revolution of the wheel?
c) Does the fish reenter the water? If so, for approximately how long?
d) After 36 seconds, at what height will the fish be?
e) Give an equation for the curve as a sine function.

a) min–to–max will be the diameter = 8 m.

6 to −2: 8

b) One revolution is One Period - 8 seconds (from t = 4 to t = 12.)

c) yes. The fish is in the water for the "yellow" parts of the sketch from ~6.75 s to 9.25 sec ⇒ 2.5 seconds in the water

d) requires a fn ⇓ next page

CA: $y = 2$

distance is 4

$a = 4$ (max of 6, CA of 2)

If $d = 4$, we being at a max (the point $(4, 6)$)

$\Rightarrow$ positive cosine with $d = 4$

$P = 2.5 \Rightarrow k = \frac{360}{P} = \frac{360}{2.5} = 144$ $\quad k = 144$

$$h(t) = 4\cos(144(t-4)) + 2$$

17. Using a graphing calculator, graph each of the following functions. Determine if the functions are periodic. If so, determine if they are sinusoidal (sinusoidal means having the basic shape of a sine or cosine function).
a) $y = x\sin x$
b) $y = \sin x - \cos x$

Ignore this question — I will give you sketches to decide if periodic or not (eg see the 1st part of the notes for U6L1)

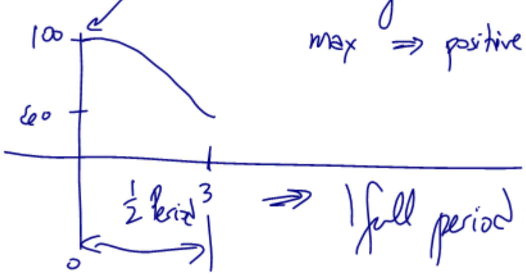
18. The size of a certain population of wild horses varies because of certain ecological pressures. In 1999, the number of horses was 100. In 2002, the population was down to 60, but in 2005, the population was back up to 100.
a) Assume that 100 was the maximum number of horses and 60 was the minimum. Write an equation that models the size of the population of horses, in terms of what year it is, starting with 1999.
b) What is the equation of the axis of this equation, and what does it represent?
c) How many horses were there in 2000?
d) How many horses should there be in 2015?

1999 means $t = 0$
2002 means $t = 3$

max = 100   1999
min = 60   2002

3 years from max to min

at "t" = 0 we begin at a max $\Rightarrow$ positive cosine with $d = 0$



$\Rightarrow$ 1 full period is 6 years

$a = \frac{max - min}{2} = \frac{100 - 60}{2} = 20$

$CA = \frac{max + min}{2} = \frac{100 + 60}{2} = 80$

$P = 6 \Rightarrow k = \frac{360}{P} \Rightarrow k = \frac{360}{6} = 60$

a) $P(t) = 20\cos(60t) + 80$

b) $y = 80$ — this is the average population of horses (between max pop & min population)

c) the year 2000 means $t = 1 \Rightarrow P(1) = 20\cos(60) + 80 = 90$ horses

d) " " 2015 " $t = 16 \rightarrow P(16) = 20\cos((60)(16)) + 80 = 70$ horses