

11U U7: Sequences and Series Happy Fun-Times Problems

Formulas:

Arithmetic Sequences

Recursive: $t_1 = a, t_n = t_{n-1} + d$

General Term $t_n = a + (n-1)d$

Geometric Sequences

Recursive: $t_1 = a, t_n = r \cdot t_{n-1}$

General Term: $t_n = a \cdot r^{n-1}$

Arithmetic Series

General: $S_n = \frac{n(2a + (n-1)d)}{2}$

Know Last Term: $S_n = \frac{n(t_1 + t_n)}{2}$

Geometric Series

General: $S = \frac{a(r^n - 1)}{(r - 1)}$

Know Last Term: $S_n = \frac{t_{n+1} - t_n}{(r - 1)}$

Practice writing clear and well written solutions using the following problems. Also - help each other learn.

1. Find the general term of the sequence: 7, 13, 19, 25, 31, ... and find t_{50} . Smile - you're doing math!

$$t_n = 7 + (n-1)(6)$$

$$t_{50} = 7 + (49)(6) = 301$$

2. The 3rd term of an arithmetic sequence is 7, and the 7th term is 23. Determine t_{20} and make someone's day a little brighter.

$$t_3 = a + 2d = 7 \quad \textcircled{1}$$

$$t_7 = a + 6d = 23 \quad \textcircled{2}$$

$$\textcircled{2} - \textcircled{1} \Rightarrow$$

$$4d = 16$$

$$\Rightarrow \boxed{d = 4}$$

$$\text{sub into } \textcircled{1} \Rightarrow a + 2(4) = 7$$

$$\begin{aligned} a + 8 &= 7 \\ \Rightarrow a &= -1 \end{aligned}$$

$$\begin{aligned} t_{20} &= a + 19d \\ &= -1 + (19)(4) \\ &= 75 \end{aligned}$$

3. Determine the number of terms in the arithmetic sequence: -45, -32, -19, -6, ..., 124. Did you hear about the constipated accountant? She couldn't budget. Did you hear about the constipated mathematician? He worked it out with a pencil - at least he didn't use a calculator to work it out...

$$a = -45$$

$$t_n = 124$$

$$\begin{aligned} d &= -32 - (-45) \\ &= +13 \end{aligned}$$

$$\Rightarrow a + (n-1)d = 124$$

$$\Rightarrow -45 + (n-1)(13) = 124$$

$$\Rightarrow 13(n-1) = 169$$

$$\Rightarrow n-1 = 13$$

$$\Rightarrow \boxed{n = 14}$$

∴ 14 terms

4. A music hall has 27 seats in the first row, 34 in the second row, 41 seats in row 3, and so on. The pattern continues in the rows of seats. The last row has 181 seats. There is an amazing vending machine in the lobby dispensing the most delicious foods you can think of like spaghetti and meatballs dinner and lemon meringue pie. Yum.

a) How many seats are in the tenth row of the hall?

b) How many seats are there in the hall?

$$\begin{aligned} a) \quad a &= 27 \\ d &= 7 \end{aligned}$$

$$\begin{aligned} t_{10} &= a + 9d \\ &= 27 + 9(7) \\ &= 90 \end{aligned}$$

\therefore The 10th row has 90 seats

\rightarrow add up the rows \Rightarrow Series - we know $t_1 \neq t_n$
 \Rightarrow use $S_n = \frac{n(t_1 + t_n)}{2}$
 \Rightarrow need n .

$$b) \quad t_n = 181 = 27 + (n-1)(7)$$

$$\Rightarrow 154 = 7(n-1)$$

$$\Rightarrow 22 = n-1$$

$$\Rightarrow \boxed{n = 22}$$

$$\begin{aligned} \therefore S_{22} &= \frac{(22)(27 + 181)}{2} \\ &= 2288 \end{aligned}$$

\therefore There are 2288 seats.

5. Write the general term of the geometric sequence: 4, -8, 16, -32, ... and find t_7 . Encourage your neighbor with a thoughtful compliment. Be nice to each other.

$$\begin{aligned} a &= 4 \\ r &= \frac{t_2}{t_1} = \frac{-8}{4} \\ &= -2 \end{aligned}$$

$$\begin{aligned} t_7 &= ar^6 \\ &= 4(-2)^6 \\ &= 256 \end{aligned}$$

6. Given a sequence with $t_1 = -34$ and a common difference between terms of $d = +13$, determine the sum of the first 250 terms. Doing so will make you a better human being. You are welcome.

know: a, d, n

$$\Rightarrow S_n = \frac{n(2a + (n-1)d)}{2}$$

$$\Rightarrow S_{250} = \frac{250(2(-34) + (249)(13))}{2}$$

$$= 396,125$$

use $S_n = \frac{n(t_1 + t_n)}{2}$

7. Tell your neighbor(s) a good joke, and then calculate the sum of the arithmetic series: -396 - 308 - 220 - 132 - ... + 836.

$$\begin{aligned} t_1 &= -396 \\ t_n &= 836 \end{aligned}$$

$$\begin{aligned} d &= t_2 - t_1 \\ &= -308 - (-396) \\ &= 88 \end{aligned}$$

$$836 = (-396) + (n-1)(88)$$

$$\Rightarrow 1232 = 88(n-1)$$

$$\Rightarrow n-1 = \frac{1232}{88}$$

$$\Rightarrow n-1 = 14$$

$$\Rightarrow n = 15$$

$$\begin{aligned} \therefore S_{15} &= \frac{15(-396 + 836)}{2} \\ &= 3300 \end{aligned}$$

need n

$$\rightarrow t_n = a + (n-1)d$$

8. Grace works in a Christmas toy factory. On the 20th day of work Grace assembled 137 toys. Grace noticed that since starting her job she has managed to increase how many toys she assembles by 3 toys per day, because she cares about all the children. What is the total number of toys Grace has assembled in the first 20 days of her work?

arithmetic with $d=3$

$t_{20} = 137$
 $d = 3$
 Use $S_n = \frac{n(t_1 + t_n)}{2}$

$t_{20} = a + 19d$
 $\Rightarrow 137 = a + 19(3)$
 $\Rightarrow a = 137 - 57$
 $\Rightarrow a = 80$

$S_n = \frac{n(t_1 + t_n)}{2}$
 $\Rightarrow S_{20} = \frac{20(80 + 137)}{2} = 2170$

\therefore Grace assembled 2170 toys in 20 days.

9. Volunteer to help someone in these trying times and determine S_{10} for the geometric series $6 + 30 + 150 + \dots$

$a = 6$
 $r = \frac{t_2}{t_1} = \frac{30}{6} \Rightarrow r = 5$

$S_{10} = \frac{6(5^{10} - 1)}{5 - 1}$
 $= 14\ 648\ 436$

don't have the last term
 \Rightarrow use $S_n = \frac{a(r^n - 1)}{r - 1}$

10. At TDChristian we have a phone list in case of an emergency closure (such as snow days). Mr. Groot is at the top of the call list and he must call the three people next on the list. Each of those people call three people also. The pattern continues so that at every level of the call list, each person is responsible to call three people. After 7 levels, how many people would be called with the news of a closure? The first snow day of a semester is like getting Christmas presents, don't you think?

Geometric.

$n = 6$
 (since Mr. Groot isn't "called")

$a = 3$ (not 1 since Mr. Groot isn't called)
 $r = 3$

we know $t_1 = 3$, $r = 3$, $n = 6$. We do not know t_6
 \Rightarrow use $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_6 = \frac{(3)(3^6 - 1)}{3 - 1}$
 $= 1092$

Series (total people called)

\therefore 1092 phone calls are made.