Name Solns______________

A∞Ω
MATH@TD

11U U7: Sequences and Series Check

K ____/8 T ____/11 C ____/3 A ____/8

Formulas: (and you may use your notes)

Arithmetic

$t_1 = a, \quad t_n = t_{n-1} + d$

$t_n = a + (n-1)d$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

Geometric

$t_1 = a, \quad t_n = r \cdot t_{n-1}$

$t_n = a \cdot r^{n-1}$

Multiple Choice

K ____/5

Identify the choice that best completes the statement or answers the question.

1. What is the 10th term of the arithmetic sequence: 1, $\overset{d=3}{4}$, 7, 10, 13, … ?
   "d"
   a. 27 [✓ c.] 28
   b. 31 d. 25

   $t_{10} = 1 + (9)(3)$
   $= 28$

2. If the first term of a sequence is $\overset{a=3}{3}$ and the common difference is $\overset{d=4}{4}$, what is the $\overset{n-1=22}{\text{23rd}}$ term in the sequence?
   a. 88 c. 95
   b. 87 [✓ d.] 91

   $t_{23} = 3 + (22)(4)$
   $= 91$

3. What is the general term of the geometric sequence: 6, $\overset{r=7}{42}$, 294, 2058,14406, …
   "r"
   a. $t_n = 6(7)^n$ [✓ c.] $t_n = 6(7)^{n-1}$
   b. $t_n = 6(6)^{n-1}$ d. $t_n = 7(6)^{n-1}$

   $t_n = a \cdot r^{n-1}$ $a = 6, r = 7$
   $= 6(7^{n-1})$

4. What is the 6th term of the geometric sequence: $\overset{a}{\frac{1}{2}}, \frac{2}{3}, \frac{8}{9}, \frac{32}{27}, \ldots$
   a. $\frac{4096}{729}$ c. $\frac{1024}{243}$
   b. $\frac{2048}{729}$ [✓ d.] $\frac{512}{243}$

   $r = \frac{t_2}{t_1}$
   $= \frac{2/3}{1/2}$
   $= \frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$

   $t_6 = \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)^5$
   $= \frac{1}{\cancel{2}}\left(\frac{\overset{512}{\cancel{1024}}}{243}\right)$

5. Determine the sum of the arithmetic series: $5 + \overset{+13}{18} + \overset{+13}{31} + 44 + \ldots +. 161$.
   [✓ a.] 1079 c. 996
   b. 1992 d. 2158

   $S_n = \frac{n(t_1 + t_n)}{2}$

   need n

   $t_n = 161 = 5 + (n-1)(13)$
   $\Rightarrow n - 1 = \frac{156}{13} = 12$
   $\Rightarrow \boxed{n = 13}$

   $S_{13} = \frac{(13)(5 + 161)}{2} = 1079$

Full Solution - Provide full and clear solutions to the following problems. You will receive a Communications grade out of 3 for how well you present your mathematical thinking.

6. What is the general term for the arithmetic sequence: 349, 321, 293, 265, …? Determine t_{30}. **K ____/3**

$a = 349$

$d = -28$

$d = t_2 - t_1$

$= 321 - 349$

$= -28$

$t_n = a + (n-1)d$

$\Rightarrow t_n = 349 + (n-1)(-28)$

$\Rightarrow t_{30} = 349 + (29)(-28)$

$= -463$

+13 +13 +13 ← arithmetic

7. Determine the number of terms in the sequence: –45, –32, –19, –6, ..., 124. **T ____/2**

$t_n = 124$ $\quad a = -45$, $d = +13$ $\quad (t_2 - t_1 = -32 - (-45) = -32 + 45 = +13)$

$t_n = a + (n-1)d$

$\Rightarrow \overset{+45}{124} = \overset{+45}{-45} + (n-1)(13)$

$\Rightarrow \frac{169}{13} = \frac{13(n-1)}{13}$

$\Rightarrow 13 = n - 1$

$\Rightarrow n = 14$ //

"h_n" 1.1cm

8. The grass on a golf course needs cut when it is 3 cm. If the height of the grass is 11 mm on Sunday and it grows at a rate of 2.5 mm per day. Determine a general term for the arithmetic sequence associated with this problem and determine on which day the grass should be cut. **A ____/3**

0.25 cm

adding ⇒ arithmetic

$a = 1.1$

$d = 0.25$

Let the general term be called h_n ← for height

$\Rightarrow h_n = a + (n-1)d$

$\Rightarrow h_n = 1.1 + (n-1)(0.25)$

We want 'n' when $h_n = 3$cm

$3 = 1.1 + (n-1)(0.25)$

$\Rightarrow 1.9 = 0.25(n-1)$

$\Rightarrow 7.6 = n - 1$

$\Rightarrow n = 8.6$

∴ About 8½ days later the grass will need to be cut.

9. What is the general term of the geometric sequence: 4, –6, 9, $-\frac{27}{2}$, ... ? **T ___/3**

'r'

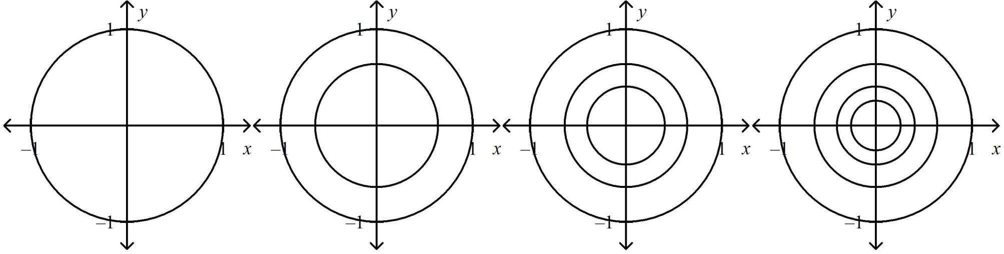
$a = 4$

$r = \frac{t_2}{t_1} = \frac{-6}{4} = -\frac{3}{2}$

$t_n = a \cdot r^{n-1}$

$\Rightarrow t_n = 4\left(-\frac{3}{2}^{n-1}\right)$

10. A sequence of inscribed circles is as follows:



Where the radius of each circle follows the sequence 1, $\frac{4}{5}$, $\frac{16}{25}$, $\frac{64}{125}$, . . .

Show that the area of the circles follows a geometric sequence by finding the general term.

Hint: $A_n = \pi(r_n)^2$ **T ____/3**

This means we want the "area sequence" to look like

1st term common ratio

$A_n = (\text{number})(\text{ratio})^{n-1}$

radius sequence

t_1 common ratio

$r_n = (1)\left(\frac{4}{5}\right)^{n-1} = \left(\frac{4}{5}\right)^{n-1}$

Area sequence

$A_n = \pi r_n^2$

$= \pi\left(\left(\frac{4}{5}\right)^{n-1}\right)^2$ ⇒ not the correct 'form' for a geometric sequence

recall an exponent fact from U4:

$(a^m)^n = (a^n)^m$

order of "exponent times exponent" doesn't matter

$= \pi\left(\left(\frac{4}{5}\right)^2\right)^{n-1}$

$= \pi\left(\frac{16}{25}\right)^{n-1}$ which is the form of a geometric sequence.

11. Determine t_{10} for the geometric sequence with $t_1 \overset{a=2}{=} 2$ and $t_4 = 54$. gives info. **T ____/3**

$t_{10} = ar^9$

known $=2$ need

$t_4 = ar^3$

$\Rightarrow 54 = (2)r^3$

$\Rightarrow 27 = r^3$

$\Rightarrow r = (27)^{\frac{1}{3}}$

$= 3$

$\therefore t_{10} = (2)(3)^9$

$= 39\,366$

12. A mason wants to start building a stone pyramid the base of which has 551 blocks and the top 26th layer has only 1. If the number of blocks on each layer follow an arithmetic sequence, how many blocks should he get? Hint - *series*. **A ____/2**

$t_1 = 551$ ← 1st term

$t_{26} = 1$ ← last term

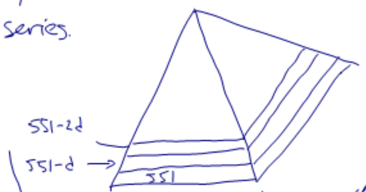
total ⇒ add up all blocks ⇒ series.

$S_{26} = \frac{(26)(551+1)}{2}$

$= 7{,}176$

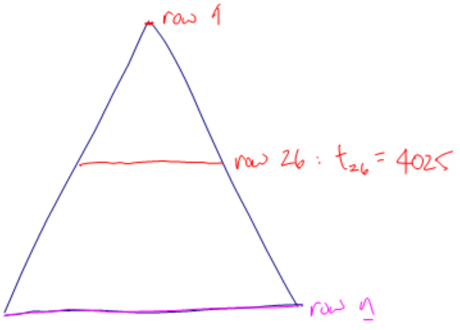
using $S_n = \frac{n(t_1 + t_n)}{2}$

$\therefore$ The pyramid will need a total of 7,176 bricks.



each layer is a term in the sequence of bricks/row.

13. A farmer has to plant seeds in a triangular field. He knows that the ***middle*** row, row 26, needs 4025 seeds, and that the last row needs 7525 seeds. If the number of seeds planted in each row follows an arithmetic series, how many total seeds does he need? **A ____/3**



We know $t_{26} = a + 25d = 4025$ ①

$t_{51} = a + 50d = 7525$ ②

the last row.

② − ① ⇒ $25d = 3500$

$\Rightarrow d = \frac{3500}{25} = +140$ sub into ① to find 'a'

$\Rightarrow a + 25(140) = 4025$

$\Rightarrow a = 525$

to find 'n' we can use some simple logic

– row 1 } 25 rows

– row 26 middle } must be 25 rows

– row n ⇒ 26 + 25 = 51.

We have $t_1 = 525$ first term

$t_{51} = 7525$ last term

⇒ use $S_n = \frac{n(t_1 + t_n)}{2}$

$\Rightarrow S_{51} = \frac{51(525 + 7525)}{2}$

$= 205{,}275$

$\therefore$ The farmer needs 205,275 seeds