AooΩ
MATH@TD

MCR3U - Chapter 7 Review

+3 +3 +3 → arithmetic

d = 3

a = 1

1. What is the 10th term of the sequence: 1, 4, 7, 10, 13, … ?
   a. 27   c. 28
   b. 31   d. 25

$t_n = a + (n-1)d$

n = 10 $\quad t_{10} = 1 + (9)(3) = 28$

2. If the first term of a sequence is 3 and the common difference is 4, what is the 23rd term in the sequence?
   a. 88   c. 95
   b. 87   d. 91

→ arithmetic

$t_{23} = a + 22d$

$= 3 + 22(4)$

$= 91$

3. The 11th term of a sequence is 24. If the first term is 374, then what is the common difference?
   a. –35   c. –34
   b. –27   d. –28

n = 11 → $t_{11} = 24$

$a = 374$ $\quad d = ?$

$t_{11} = a + 10d$

$24 = 374 + 10d$

→ $-350 = 10d$

$\Rightarrow d = -35$

-875 -875 -875 arithmetic!

4. Determine the number of terms in the sequence: 5240, 4365, 3490, 2615, ..., –2635
   a. 8   c. 11
   b. 10   d. 9

'a' $\quad t_n$

$d = -875$

$t_n = a + (n-1)d$

$\Rightarrow -2635 = 5240 + (n-1)(-875)$

$\Rightarrow -7875 = (n-1)(-875) \quad \div(-875)$

$\div -875$

$\Rightarrow (n-1) = 9 \Rightarrow \boxed{n = 10}$

5. The 13th term of a sequence is 8, and the 17th term is 8. What is the 25th term?
   a. 4   c. 8
   b. 25   d. 0

no change

_, _, _, _, _, _, _, _, _, _, _, _, 8, _, _, _, 8, _

d = 0 or r = 1

$\boxed{t_{25} = 8}$ no change!

6. Determine the general form for the arithmetic sequence whose 5th term is 10 and consecutive terms decrease by 4.

a. $22 - 4(n-1)$ c. $26 - 4(n-1)$
b. $-2 + 4(n-1)$ d. $-6 + 4(n-1)$

$\Rightarrow d = -4$

need a

$t_5 = a + 4d$

$\boxed{t_n = a + (n-1)d}$

$\Longrightarrow 10 = a + 4(-4)$

$\Longrightarrow 10 = a - 16 \Rightarrow a = 26$

$\therefore t_n = 26 + (n-1)(-4)$ c

7. What is the general term of the sequence: 6, 42, 294, 2058, 14406, … $\Rightarrow$ geometric w/ $r = 7$, $a = 6$

$\times 7$ $\times 7$ $\times 7$

a. $t_n = 6(7)^n$ c. $t_n = 6(7)^{n-1}$
b. $t_n = 6(6)^{n-1}$ d. $t_n = 7(6)^{n-1}$

$t_n = ar^{n-1}$

$\Rightarrow t_n = (6)(7^{n-1})$

8. What is the 6th term of the sequence: $\frac{1}{2}, \frac{2}{3}, \frac{8}{9}, \frac{32}{27}, \ldots$

$\times 4/3$ $\times 4/3$

a. $\frac{4096}{729}$ c. $\frac{1024}{243}$
b. $\frac{2048}{729}$ d. $\frac{512}{243}$

geometric or arithmetic?

$\frac{t_2}{t_1} = \frac{2/3}{1/2} = \frac{4}{3}$, $\frac{t_3}{t_2} = \frac{\frac{8}{9}}{\frac{2}{3}} = \frac{8}{9} \times \frac{3}{2} = \frac{4}{3}$

geometric w/ $r = \frac{4}{3}$

$t_6 = ar^5$

$= \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)^5 = \frac{1}{2}\left(\frac{1024}{243}\right) = \frac{512}{243}$ d

9. If the 9th term in a geometric sequence is 45 927 and the common ratio is 3, then what is the first term?

a. 7 c. 6
b. 8 d. 9

$r = 3$

$t_9 = 45927$

$t_9 = ar^8$

$\Rightarrow 45927 = a(3)^8$

$\Rightarrow 45927 = a(6561)$

$\Rightarrow a = \frac{45927}{6561}$

$= 7$.

10. If the 8th term of a sequence is 256, and the 5th term of the sequence is 32, what is the common ratio?

geometric

a. 2 c. -2
b. 4 d. $\frac{1}{2}$

known info: $t_8 = 256$, $t_5 = 32$

$t_n = ar^{n-1}$

$\Rightarrow \boxed{t_8 = ar^7 \quad \& \quad t_5 = ar^4}$

nice trick for geometric sequences

$\frac{t_8}{t_5} = \frac{\cancel{a}r^7}{\cancel{a}r^4} \Rightarrow \frac{256}{32} = r^3$

$\Rightarrow r^3 = 8$

$\Rightarrow r = 8^{\frac{1}{3}}$

$= 2$

11. How many terms are in the geometric sequence: 2, 6, 18, 54, ..., 118 098 ← t_n

a. 8
b. 9
c. 10
d. 11

$r = \frac{t_2}{t_1} = \frac{6}{2} = 3$

$r = 3$

$t_n = ar^{n-1}$ need to find n $a = 2$

$\Rightarrow 118\,098 = 2(3)^{n-1}$ $(\div 2)$ both sides to get the exponential alone.

$\Rightarrow 59\,049 = 3^{n-1}$ use "log" to get the "n-1" out of the exponent

$\Rightarrow \log(59049) = (n-1)\cdot\log(3)$ $\div \log(3)$

$\Rightarrow \frac{\log(59049)}{\log(3)} = n-1 \Rightarrow n-1 = 10$

$\Rightarrow n = 11$ $\therefore$ 11 terms!

12. Determine the sum of the arithmetic sequence: $\overset{t_1}{5} \overset{+13}{+} \overset{t_2}{18} \overset{+13}{+} \overset{t_3}{31} + \overset{t_4}{44} + \ldots + 161$. ← t_n!

a. 1079
b. 1992
c. 996
d. 2158

$S_n = \frac{n(2a + (n-1)d)}{2}$

$a = 5$

$d = +13$

$n = ?$

↳ we need 'n'

use $t_n = 161$

$t_n = a + (n-1)d$

$\Rightarrow 161 - 5 = 5 - 5 + (n-1)(13)$

$\Rightarrow 156 \div 13 = 13(n-1) \div 13$

$\Rightarrow 12 = n-1$

$\Rightarrow \boxed{n = 13}$

$\Rightarrow S_{13} = \frac{13(2(5) + (12)(13))}{2}$

$(13-1)$

$= \frac{13(10 + 156)}{2} = 1{,}079$

13. Calculate the sum of the first 15 terms of an arithmetic sequence with third term 14 and common difference –9.

$n = 15$!

a. –585
b. –465
c. 434
d. –824

$t_3 = 14$

$d = -9$

$$S_n = \frac{n(2a + (n-1)d)}{2}$$

use t_3 to find a

$$t_3 = a + 2d \quad (n-1)$$

$$\Rightarrow 14 = a + 2(-9)$$

$$14 = a - 18$$

$$\Rightarrow a = 32$$

$n = 15$
$d = -9$
$a = ?$
need a

$$\Rightarrow S_{15} = \frac{15\left(2(32) + (14)(-9)\right)}{2}$$

$$= -465$$

S_n n! d! $t_3 = 8$ $t_8 = 143$ ⇐ given info.

14. Calculate the sum of the first 20 terms of an arithmetic sequence with 3rd term 8 and 8th term 143.

a. 8420
b. 5400
c. 4210
d. 4480

don't know these!

$$S_n = \frac{n(2a + (n-1)d)}{2}$$

2 unknowns ⇒ we need 2 bits of info to find them!!

use t_3 & t_8 to find a & d

$$t_3 = a + 2d \qquad t_8 = a + 7d$$

$$\underbrace{\Rightarrow 8 = a + 2d \;①\qquad \Rightarrow 143 = a + 7d \;②}_{\text{system of linear eq'ns}}$$

$$② - ① \Rightarrow 135 = 5d$$

$$\Rightarrow \boxed{d = \frac{135}{5} = 27}$$

sub into ① for 'a'

$$8 = a + 2(27)$$

$$8 - 54 = a$$

$$\Rightarrow \boxed{a = -46}$$

$$\Rightarrow S_{20} = \frac{20\left(2(-46) + 19(27)\right)}{2}$$

$$= 4210$$

$n=26$ $\quad a=452$

15. Determine S_{26} for the series: –452 – 396 – 340 – 284 – ...

a. 6448
b. 7176
c. 12298
d. 6200

$$S_n = \frac{n(2a+(n-1)d)}{2}$$

$$\Rightarrow S_{26} = \frac{26(2(-452)+(25)(56))}{2}$$

$$= 6448$$

Q: Arithmetic or Geometric

⇒ compare terms to see if there is a ratio or a difference!

$$t_2 - t_1 = -396-(-452)$$
$$= 56$$
$$t_3 - t_2 = -340-(-396)$$
$$= 56$$

common difference of 56 ⇒ arithmetic!

$n=15$ $\quad t_5 = 4$

16. Determine the sum of the first 15 terms of the arithmetic series whose 5th term is 4 and consecutive terms increase by 3. $d=3$

a. 195
b. 285
c. 182
d. 169

use t_5 to find a "n-1"

$$t_5 = a + 4d$$
$$\Rightarrow 4 = a + 4(3)$$
$$4 = a + 12$$
$$\Rightarrow a = -8$$

$$S_n = \frac{n(2a+(n-1)d)}{2}$$

we need 'a'

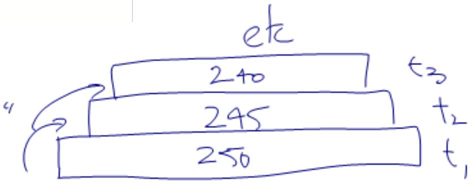
$$\Rightarrow S_{15} = \frac{15(2(-8)+14(3))}{2} = 195$$

17. A brick layer building a trapezoidal wall starts with a base of 250 bricks and decreases the number of bricks by 5 each layer up. How many bricks does he need to make the wall 26 layers high?

a. 4550
b. 4875
c. 4688
d. 5200

total ⇒ series $\quad n=26$

"d = -5"



Given info: $a = 250$
$d = -5$
$n = 26$

$$S_n = \frac{n(2a+(n-1)d)}{2}$$

$$\Rightarrow S_{26} = \frac{26(2(250)+25(-5))}{2} = 4875 \text{ bricks}$$

Contractors can actually use formulas like this to know how much material to order for a job!

common ratio! ×5 ×5 ×5

18. Determine the sum of the geometric series 3 + 15 + 75 + 375 + ... + 46 875.

a. 58 593
b. 11 718
c. 19 531
d. 292 968

$r = 5$

$a = 3$

Two solns

① $S_n = \frac{a(r^n - 1)}{r-1}$ we need n! use $t_n = 46875$

$t_n = ar^{n-1}$

$\Rightarrow 46875 = 3(5)^{n-1}$ ÷3

$15625 = 5^{n-1}$ use log.

$\Rightarrow \frac{\log(15625)}{\log(5)} = \frac{(n-1)\cdot\log(5)}{\log(5)}$

$\Rightarrow n-1 = \frac{\log(15625)}{\log(5)} = 6$

$\Rightarrow n = 7$

$\Rightarrow S_7 = \frac{3((5)^7 - 1)}{5-1} = 58593$

$\boxed{\text{OR}}$ ② use

(we know r & the last term!)

$S_n = \frac{t_{n+1} - t_1}{r-1}$, where

$t_{n+1} = (t_n)(r)$

$= (46875)(5)$

$= 234375$

$\Rightarrow S_n = \frac{234375 - 3}{5-1}$

$= 58593$

"unknown"

$n=6$ $t_3 = 6$ $r = 2/3$

19. Calculate the sum of the first 6 terms of the geometric series with 3rd term 6 and common ratio $\frac{2}{3}$.

a. $\frac{1330}{243}$
b. $\frac{211}{6}$
c. $\frac{665}{18}$
d. $\frac{665}{729}$

(Note: $\frac{665}{18} = 36.9\dot{4}$ by calculator)

use t_3 to find a

$S_n = \frac{a(r^n - 1)}{r-1}$ (need a)

$t_3 = ar^2$

$\Rightarrow 6 = a(2/3)^2$

$6 = a\left(\frac{4}{9}\right)$

$\Rightarrow a = \frac{6}{4/9}$

$= \frac{6^3}{1} \times \frac{9}{4_2}$

$= \frac{27}{2}$

$\Rightarrow S_6 = \frac{\frac{27}{2}\left(\left(\frac{2}{3}\right)^6 - 1\right)}{\frac{2}{3} - 1} = 36.9\dot{4}$

calculator answer is ok on test

MC answer

$= \frac{\frac{27}{2}\left(\frac{64}{729} - 1\right)}{\frac{2}{3} - \frac{3}{3}}$

$= \frac{\frac{27}{2}\left(\frac{64}{729} - \frac{729}{729}\right)}{-1/3}$

$= \frac{\frac{27}{2}\left(-\frac{665}{729}\right)}{-\frac{1}{3}}$

$= \frac{\cancel{27}^{1}}{2} \times \frac{-665}{\cancel{729}_{\cancel{243}\,9}} \times \frac{-\cancel{3}^{1}}{1}$

$= \frac{665}{18}$

$n=7$ $r=?$ $a=2$ $t_4=250$

20. Calculate the sum of the first 7 terms of the geometric series with first term 2 and 4th term 250.
    a. 7812 c. 19 531
    b. 195 312 d. 39 062

$S_n = \frac{a(r^n-1)}{r-1}$ — we need 'r' → use t_4 to find r

$t_4 = ar^3$

$\Rightarrow 250 = 2(r^3)$

$\Rightarrow 125 = r^3$

$125 = 5^3!$ $\Rightarrow 5^3 = r^3 \Rightarrow \boxed{r=5}$

$\Rightarrow S_7 = \frac{2(5^7-1)}{5-1}$

$= 39,062$

$t_3 = 5$ $t_6 = 40$

21. Calculate the sum of the first 8 terms of the geometric series whose 3rd term is 5 and 6th term is 40.
    a. $\frac{1275}{4}$ c. $\frac{1275}{2}$
    b. $\frac{635}{4}$ d. $\frac{635}{2}$

$n=8$

$t_3 = ar^2$, $t_6 = ar^5$

$S_n = \frac{a(r^n-1)}{r-1}$, we need a & r

(2 things needed → use 2 bits of info!)

$\Rightarrow$ common trick for geometric ÷ terms

$\frac{t_6}{t_3} = \frac{\cancel{a}r^5}{\cancel{a}r^2}$

$\Rightarrow \frac{40}{5} = r^3$

$\Rightarrow r^3 = 8$

$\Rightarrow \boxed{r = 8^{1/3} = 2}$

sub into t_3 to find a

$t_3 = ar^2$

$\Rightarrow 5 = a(2^2)$

$\Rightarrow 5 = 4a$

$\Rightarrow \boxed{a = \frac{5}{4}}$

$\Rightarrow S_8 = \frac{(\frac{5}{4})(2^8-1)}{2-1}$

$= \frac{5}{4}(256-1)$

$= \frac{1275}{4}$

$n=7$ $\quad t_3 = 4$

22. Determine the sum of the first 7 terms of the geometric series whose 3rd terms is 4 and consecutive terms increase by a factor of $\frac{3}{2}$. $\quad r = \frac{3}{2}$

a. $\frac{2059}{64}$
b. $\frac{665}{2}$
c. $\frac{665}{18}$
d. $\frac{2059}{36}$

$S_n = \frac{a(r^n - 1)}{r-1}$ need a → use t_3 to find a

$t_3 = ar^2$

$\Rightarrow 4 = a\left(\frac{3}{2}\right)^2$

$\Rightarrow 4 = \frac{9}{4}a \Rightarrow a = \frac{4}{\frac{9}{4}} = \frac{4}{1} \times \frac{4}{9}$

$\boxed{\Rightarrow a = \frac{16}{9}}$

$$S_7 = \frac{\left(\frac{16}{9}\right)\left(\left(\frac{3}{2}\right)^7 - 1\right)}{\frac{3}{2} - 1}$$

$$= \frac{\frac{16}{9}\left(\frac{2187}{128} - \frac{128}{128}\right)}{0.5}$$

$$= \frac{16}{9}\left(\frac{2059}{128}\right) \times 2 = \frac{2059}{36}$$

$r=3$ $\quad a=2$

23. A population of rabbits if left unchecked will triple every three months. If there are initially only 2 rabbits, how many will there be in a year and a half?

a. 1458
b. 486
c. 729
d. 2187

in a year and a half there are 6 3 month periods
→ $n=6$

(kind of a nasty question)

Sequence:

$2 \xrightarrow{1} 6 \xrightarrow{2} 18 \xrightarrow{3} 54 \xrightarrow{4} 162 \xrightarrow{5} 486 \xrightarrow{6} 1458$

t_7 after 6, 3 month, periods

$t_7 = ar^6$

$= (2)(3^6) = 1458$