Functions 11 – Extra Exam Review Problems

Unit 1 – Introduction to Functions

1. Determine the domain and range of each function:
   a. $f(x) = -2(x-3)^2 + 5$
   b. $g(x) = \sqrt{x+3}$
2. For the two functions in #1 determine: a) $f(0)$ b) $g(9)$
3. On a set of axes sketch a relation which is not a function. Explain why your sketch does not represent a function.
4. Given the function state the parent function and all transformations which would turn the parent function into the given (transformed) function: $f(x) = -2\sqrt{3x-6}+1$. On the same set of axes sketch the parent function and the transformed function.

Unit 2 – Rational Expressions

1. Factor the following:
   a. $x^2 + 5x + 6$
   b. $16x^4 - 81$
   c. $4p^2 - 12pq + 9q^2$
2. Have a snack so delicious that it's ridiculous.
3. Simplify. Be sure to state all restrictions on the variable.
   a. $\dfrac{x^2 + 2x - 8}{(x+4)(x+1)}$
   b. $\dfrac{2x-1}{x^2-x-6} \times \dfrac{2x^2+5x+2}{4x^2-1}$
   c. $\dfrac{x^2-25}{x^2-10x+25} \div \dfrac{2x^2+9x-5}{3x^2-13x-10}$
   d. $\dfrac{3x}{4x^2-9} - \dfrac{4x}{6x^2+13x+6}$ (be careful with your signs when simplifying!)

Unit 3 – Quadratics

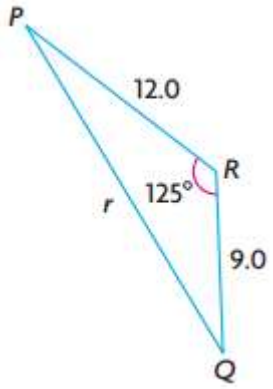
1. Given $f(x) = -5x^2 + 20x + 1$
   a. Determine the equation of the AoS
   b. The max/min value. Which is it, a max or a min, and why?
   c. How many zeros does the function have (there are at least two acceptable answers here)
   d. Determine the zeros using any method of your choice.
   e. Express the function in vertex form. State the domain and range of the quadratic.
2. Simplify:
   a. $3\sqrt{12} - 6\sqrt{75}$ b. $\sqrt{3}\left(2\sqrt{5} + 5\sqrt{2}\right) + \sqrt{24}$ c. $\left(2 + \sqrt{5}\right)\left(1 - \sqrt{10}\right)$
3. Determine the equation, in zeros form, of a quadratic function which has the following characteristics:
   a. $y_{\text{int}} = -2$
   b. The quadratic has two zeros: $x = -1,\ x = 3$
4. Smile – you're doing math!
5. Sketch the graph of the function $f(x) = x^2 + 2x - 3$. Label the vertex, the zeros, the y-intercept and the AoS.
6. Solve the equations:
   a. $2x^2 - 3x = 5$
   b. $3x^2 + x - 1 = 5x + 1$
7. What is the discriminant, and what does it do, and how does it do what it does?
8. The height, $h(t)$, in meters, of the trajectory of a baseball is given by $h(t) = 1.4 + 22.4t - 4.9t^2$. Determine the maximum height of the baseball and the time when that height is achieved.

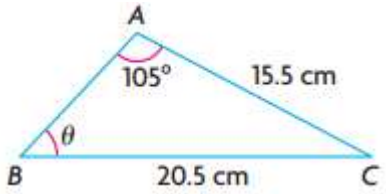
Unit 4 – Exponential Functions

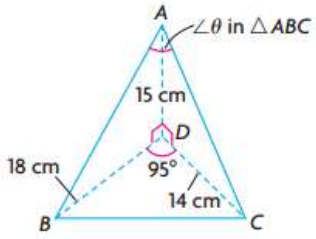
1. Evaluate. Express as a fraction and rounded to decimal places: $\left(-\frac{8}{27}\right)^{-\frac{4}{3}}$.
2. Simplify, using only positive exponents in your answer:
   a. $\left(-2x^2y^{-1}\right)^3\left(3xy^2\right)^2$
   b. $\sqrt[4]{\frac{32x^{-2}}{(2x)^{-3}}}$
   c. $\frac{\left(3p^2q^{-3}\right)^{-2}}{9p^{-6}q^{10}}$

3. A small city had a population of 106,000 in the year 2001. If the city was experiencing an average annual growth rate of 1.33% determine the population of the city in the year 2011.
4. As hard as it may be to believe, God loves you.
5. How are exponential functions similar to the formulas of financial mathematics?
6. At 2:30pm a colony of 50 million viruses make a home in your body. (Thanks to the student who sneezed on you earlier in the day. Ew.) The doubling period of the virus is 3.5 hours. What is the population of the bacteria colony when you wake up the next day at 6am?

Unit 5 – Trigonometric Ratios

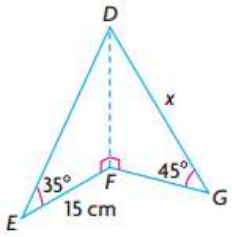
1. The terminal arm of some angle of rotation, θ, ends at the point $P(-3,4)$. Sketch the angle of rotation and determine the six trigonometric ratios for θ.
2. Determine angle(s) x, where $0^\circ \le x \le 360^\circ$, for:
   a. $\sin(x) = -\frac{\sqrt{3}}{2}$
   b. $\sec(x) = \sqrt{2}$
   c. $\cos(x) = 0.1253$
3. Prove: a) $1+\cot^2(\alpha) = \csc^2(\alpha)$ b) $\csc(\theta)\sec(\theta) = \cot(\theta)+\tan(\theta)$
4. A building of height h is observed from two points, P and Q, which are $75m$ apart. The angle of elevation from point Q is 37°, and 44° from point P. Draw a picture representing the situation and determine the height, h, of the building.
5. Stop and smell the flowers. And look at the sky.
6. Solve the triangles:
   a.


   b.




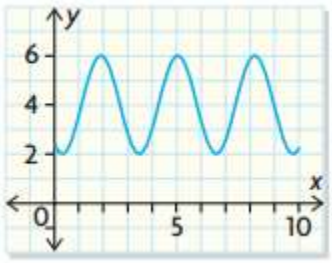
7. Determine the value of x, to the nearest hundredth, in the diagram to the right:

8. Determine the value of θ, to the nearest degree, in the diagram to the left:



Unit 6 – Sinusoidal Functions

1. Sketch two cycles of the function $f(\theta)=3\sin\left(2\theta-120^\circ\right)+1$. Also, state:
   a. The range of $f(\theta)$
   b. The period of the function
   c. The phase shift
2. Determine two equations, one a Sine and the other a Cosine, which describe the sketch to the right:



3. It's not good to sit for too long. It's time for the 6th Chapter stretch. Stand up sit down, stand up sit down, stand up sit down, stand up sit down. Were you a cosine or a sine?
4. An oscilloscope is measuring the current (in *Amps*) from an AC source (Alternating Current). At $t=0$ the oscilloscope records a max current of $3.2A$. At $t=\frac{1}{120}$ seconds the oscilloscope records the first minimum current of $-3.2A$. Determine an equation of a function which will describe the current in terms of time.

Unit 7 – Sequences and Series

1. The first term of an arithmetic sequence is 7. If $t_{10}=-24.5$, determine:
   a. The general term for the sequence, t_n, and the recursive formula.
   b. t_{25}
   c. S_{25}
2. Determine the recursive formula and the general term, t_n, for the geometric sequence: $0.8,\ 2.4,\ 7.2,\ldots$
   a. Determine t_9
   b. Help someone. Helping others is good.
   c. Determine S_{11}

Unit 8 – Financial Mathematics

1. One day you're ambling alongside a sunny river when you suddenly trip over a paper sack. Inside the sack you find two things: a lollipop stick with the remnants of a grape lolli, and $67 500. Being a good citizen you take the sack of money to the nearest police station. Along the way you make certain that the lollipop stick finds its way into a trash bin. After filling out the paperwork for the found money you go home and wait. 30 days. It's a lot of waiting, but you fill the time in the best way possible. You do some math. Finally the 30 days are up and the money, never being claimed, is yours. You take the money straight to the bank. You do not pass go. You do not collect $200 (no need to be greedy!). You deposit the $67 500 in an account paying 1.2%/a compounded quarterly. How much money will be in the account after 7.5 years?

2. Miriam borrows $14 500 to buy a car. The finance company charges 2.4%/a compounded monthly. If Miriam plans to pay the loan off in 4 years, how much will she have to pay the finance company every month?

3. At the age of 25 you open a savings account which pays 0.8%/a compounded monthly. Every month, faithfully, you manage to deposit $1500 for 40 years. What is the value of the account when you are 65? How much interest did you earn?

4. Study well for the exam. Get a good sleep on the night before the exam!