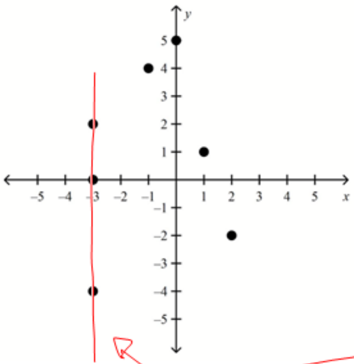
Exam Review (more problems)

Here are a few more review problems. Solutions to be posted Saturday. Note that you may have seen some of these problems on previous review or previous tests (there are only so many problems to go around...)

1. What are the domain and range of the graph? Is the graph a function?



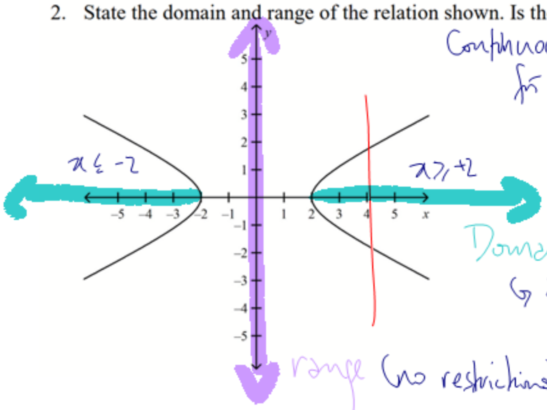
Discrete fn - just list the x & y-values

$D: \{-3, -1, 0, 1, 2\}$

$R: \{-4, -2, 0, 1, 2, 4, 5\}$

This sketch fails the VLT $\Rightarrow$ not a fn

2. State the domain and range of the relation shown. Is the relation a function?



Continuous fn

$D: \{x \in \mathbb{R} \mid x \leq -2,\ x \geq 2\}$

$R: \{y \in \mathbb{R}\}$

This is not a fn because it fails the VLT.

3. A rectangular aquarium has length $(x + 10)$, width $(x + 4)$, and height $(x + 6)$. Determine a simplified function that represents the volume of the aquarium.

$V = l \times w \times h = (x+10)[(x+4)(x+6)]$

$= (x+10)[x^2 + 6x + 4x + 24]$

$= (x+10)(x^2 + 10x + 24)$

$= x^3 + 10x^2 + 24x + 10x^2 + 100x + 240$

$= x^3 + 20x^2 + 124x + 240.$

4. Simplify $\frac{4c + 16}{5c} \div \frac{c + 4}{15c^3}$ and state any restrictions on the variables.

$= \frac{4\cancel{(c+4)}}{\cancel{5c}} \times \frac{\cancel{15c^3}}{\cancel{c+4}}$ $\quad c \neq 0, -4$

$= \frac{12c^2}{1}$

$= 12c^2$

5. Simplify $\frac{4 - x}{3x^2 - 4x - 4} \div \frac{5x - 20}{6x^2 - 17x + 10}$ and state any restrictions on the variables.

$= \frac{\cancel{4-x}}{(3x+2)\cancel{(x-2)}} \times \frac{(6x-5)\cancel{(x-2)}}{5\cancel{(x-4)}}$ $\quad x \neq -\frac{2}{3}, +2, 4, \frac{5}{6}$

$= \frac{-(6x-5)}{5(3x+2)}$

6. A quadratic function has these characteristics:
$x = 1$ is the equation for the axis of symmetry. verfex
$x = 2$ is an x-intercept. point $(2,0)$
$y = 2$ is the maximum value. vertex
Determine the y-intercept of this parabola. vertex $(1,2)$

$f(x) = a(x-h)^2 + k$

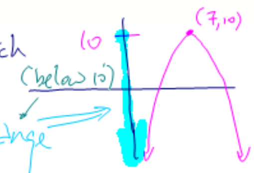
using vertex form. (use $(2,0)$ to find 'a')

$f(x) = a(x-1)^2 + 2$
$0 = a(2-1)^2 + 2$
$\rightarrow -2 = a(1) \Rightarrow a = -2$
$\Rightarrow \boxed{f(x) = -2(x-1)^2 + 2}$

$\rightarrow$ $y$int $\Rightarrow x = 0$
$f(0) = -2(0-1)^2 + 2$
$= -2(1) + 2$
$= 0$
$\therefore$ $y$int is $(0,0)$

7. Does the parabola for the function $f(x) = -(x-7)^2 + 10$ open up or down? What is the range? Explain your answer.

$f(x)$ opens down.
vertex $(7,10)$ $\rightarrow$ basic sketch



$R_f : \{ f(x) \in \mathbb{R} \mid f(x) \leq 10 \}$
below 10.

8. The cost function for a container company is $C(x) = 10x + 30$ and the revenue function is $R(x) = -x^2 + 24x$, where x is the number of containers sold, in thousands. Determine the profit function for the number of containers sold. Then determine the number of containers sold that maximizes profit. need the vertex

profit = revenue − cost
(money in) (money out)

$\Rightarrow P(x) = (-x^2 + 24x) - (10x + 30)$

$\Rightarrow \boxed{P(x) = -x^2 + 14x - 30}$ (standard form)

$\rightarrow$ to find the vertex $\rightarrow$ find the AoS
or best for standard form

$AoS = -\frac{b}{2a} = -\frac{14}{2(-1)} = +7$

or avg. of zeros for zeros form

$\Rightarrow$ vertex is $(7, P(7))$ $= (7, 19)$

$P(7) = -(7)^2 + 14(7) - 30$
$= 19$

(Vertex is $(AoS, P(AoS))$)
max or min value

$\therefore$ 7000 containers will give a max. profit of \$19 (thousand / hundred thous. / million? info not given)

9. Travis and Laura are rock climbing. Travis throws a spike to Laura. The function $h(t) = -5t^2 + 20t + 110$ models the height of the spike in metres above the ground at time t. Laura is 135 m above the ground. Did Travis' throw reach Laura? Explain your answer.

$\hookrightarrow$ i.e., did the spike Travis threw reach a height of 135
ie) can $h(t) = 135$?

$135 = -5t^2 + 20t + 110$

$\Rightarrow 5t^2 - 20t + 25 = 0$ $\div 5$

$\rightarrow t^2 - 4t + 5 = 0$
does not factor
$\Rightarrow$ quadratic formula
(but, I suspect "no solution")

$\rightarrow$ so I will check the discriminant 1st
$b^2 - 4ac$
$= (-4)^2 - 4(1)(5)$
$= -4 < 0$

$\rightarrow \therefore$ no soln
$\therefore$ No, the spike does not reach Laura

10. Simplify.
$3\sqrt{12} + \sqrt{24} - 2\sqrt{36}$

$= 3\sqrt{4 \times 3} + \sqrt{4 \times 6} - 2(6)$

$3(\sqrt{4})\sqrt{3} = 3(2)\sqrt{3}$

$= 6\sqrt{3} + 2\sqrt{6} - 12$

11. Simplify $(7 + \sqrt{50})(-9 - \sqrt{32})$.

$= (7 + \sqrt{25 \times 2})(-9 - \sqrt{16 \times 2})$

$= (7 + 5\sqrt{2})(-9 - 4\sqrt{2})$

$= -63 - 28\sqrt{2} - 45\sqrt{2} - 20(2)$

$\rightarrow = -73\sqrt{2} - 103$.

12. Simplify the expression. Express your answer with positive exponents. Explain each of your steps.
$\hookrightarrow$ the math is your explanation.

$$\left(\frac{(4x^6)^3 (4y^{-8})}{(2x)^4 (12y^3)^2} \right)^{\frac{1}{2}}$$

$$= \left(\frac{(4^3 x^{18})(4^1 y^{-8})}{(2^4 x^4)(12^2 y^6)} \right)^{\frac{1}{2}}$$

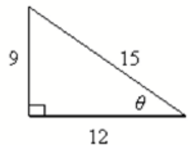
$$= \left(\frac{4^4 x^{18-4} y^{-8-6}}{(2^4)(12^2)} \right)^{\frac{1}{2}}$$

$$= \left(\frac{4^4 x^{14} y^{-14}}{(2^4)(12^2)} \right)^{\frac{1}{2}}$$

$$= \frac{4^2 x^7 y^{-7}}{(2^2)(12)}$$

$$= \frac{16x^7}{48y^7} = \frac{x^7}{3y^7}$$

13. Given the following triangle, state the six trigonometric ratios for $\angle\theta$.

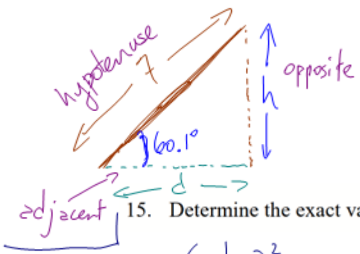


$\sin(\theta) = \frac{9}{15} = \frac{3}{5}$ $\quad \csc(\theta) = \frac{5}{3}$

$\cos(\theta) = \frac{12}{15} = \frac{4}{5}$ $\quad \sec(\theta) = 5/4$

$\tan(\theta) = \frac{9}{12} = 3/4$ $\quad \cot(\theta) = 4/3$

14. The base of a 7 m log rests against the ground. It ramps up to a branch in a tree at an angle of elevation of 60.1°.
a) Calculate the height of the branch to the nearest tenth of a metre.
b) What is the distance from the base of the tree to the base of the log?



$\sin(60.1) = \frac{h}{7}$

$\Rightarrow h = 7 \cdot \sin(60.1)$

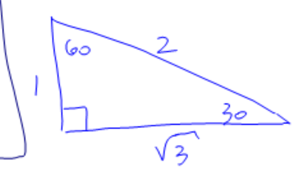
$= 6.07\ m$

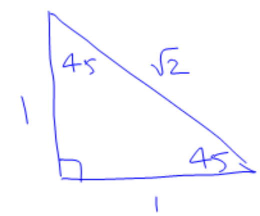
$\cos(60.1) = \frac{d}{7}$

$\Rightarrow d = 7 \cdot \cos(60.1)$

$= 3.49\ m.$

Special Δ's





15. Determine the exact value of $\frac{\cos^2 45°}{\sin 30°} \times \tan 60°$.

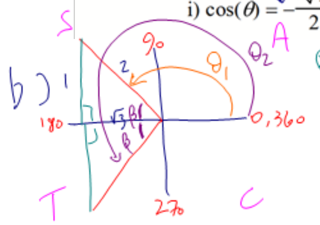
$$= \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{\left(\frac{1}{2}\right)} \times \frac{\sqrt{3}}{1} = \frac{\frac{1}{2}}{\frac{1}{2}} \times \sqrt{3}$$

$$= \sqrt{3}$$

16. Determine:
a) the trig ratios **exactly** for the following: i) sin(225) ii) cos(120)
b) the angles (exactly for i) rounded to one decimal place for ii))

Q2, Q3 $\quad$ i) $\cos(\theta) = -\frac{\sqrt{3}}{2}$ $\qquad$ ii) $\sin(\theta) = -0.2345$ Q3, Q4

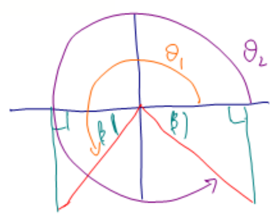
b)



$\beta = 30°$

$\theta_1 = 180 - 30$
$= 150°$

$\theta_2 = 180 + 30$
$= 210°$



$\sin(\theta) = -0.2345$

$\Rightarrow \sin(\beta) = +0.2345$

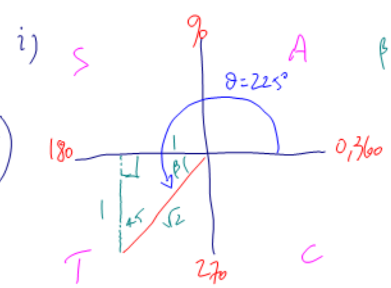
$\Rightarrow \beta = \sin^{-1}(0.2345)$

$= 13.6°$

$\theta_1 = 180 + 13.6 = 193.6°$ $\quad \theta_2 = 360 - 13.6 = 346.4°.$

a)

i)



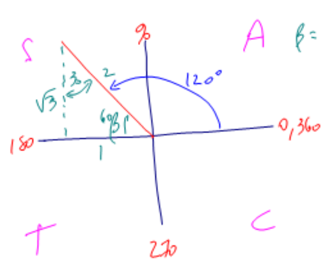
$\beta = 225° - 180°$
$= 45°$

$\sin(225°)$

$= -\frac{1}{\sqrt{2}}$

$\left(= -\frac{\sqrt{2}}{2}\right)$

ii



$\beta = 180 - 120$
$= 60°$

$\cos(120°)$

$= -\frac{1}{2}$

17. Finish the proof of the following identity. (note - on the exam you will have one identity to fully prove)

$$1 = \frac{(\sin^4 x - \cos^4 x)}{\tan x \sin x \cos x - \cos^2 x}$$

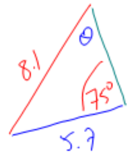
R.S.: $\frac{(\sin^4 x - \cos^4 x)}{\tan x \sin x \cos x - \cos^2 x}$

$$= \frac{(\sin^2 x - \cos^2 x)\cancel{(\sin^2 x + \cos^2 x)}^{1}}{\frac{\sin x}{\cancel{\cos x}} \sin x \cancel{\cos x} - \cos^2 x}$$

$$= \frac{\sin^2(x) - \cos^2(x)}{\sin^2(x) - \cos^2(x)}$$

$$= 1 = LS \ \square$$

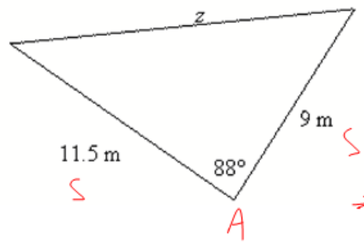
18. A triangular plot of land is enclosed by a fence. One side of the fence is 8.1 m long with an opposite angle of 75°. An adjacent side of the fence is 5.7 m long with an opposite angle of θ.
a) Make a sketch of the situation.
b) Determine θ to the nearest degree.

a) 

b) $\frac{\sin(\theta)}{5.7} = \frac{\sin(75)}{8.1} \Rightarrow \sin(\theta) = \frac{(5.7)(\sin(75))}{8.1} \Rightarrow \theta = \sin^{-1}\left(\frac{(5.7)(\sin(75))}{8.1}\right) \doteq 43°$

42.8

19. Determine z to the nearest tenth of a metre.



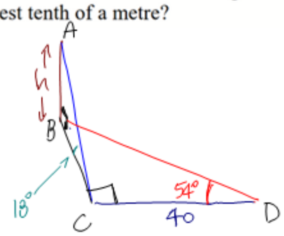
$\Rightarrow$ cosine law

$z^2 = (11.5)^2 + (9)^2 - 2(11.5)(9)\cos(88)$

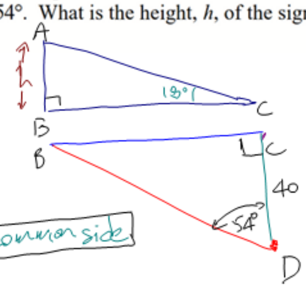
$\Rightarrow z = \sqrt{\quad " \quad}$

$\doteq 14.4$ m

20. Jake wants to know the height of a sign across a road. He stands directly across from the sign and notices the angle of elevation to the top of the sign is 18°. Jake then walks 40 m parallel to the road and observes the angle between the base of the sign and Jake's previous spot is 54°. What is the height, h, of the sign to the nearest tenth of a metre?



separating Δ's



note: BC is a $\boxed{\text{common side}}$

3D-TRIG.

In ΔBCD $\tan(54) = \frac{BC}{40}$

$\Rightarrow BC = (40)(\tan(54)) = 55.06$ m

In ΔABC $\tan(18) = \frac{h}{BC}$

$\Rightarrow h = (BC)\cdot\tan(18) = (55.06)\cdot\tan(18)$

$= 17.9$ m.

21. For the following table of data, determine an equation of a sine function that satisfies the given data. (note - be sure you know all of the transformations of sinusoidal functions) ~~You might have to sketch one)~~

x	–120°	–60°	0°	60°	120°	180°	240°
y	1.5	3	1.5	0	1.5	3	1.5
	CA	max	CA	min	CA	max	CA

neg sine

$a = \frac{max - min}{2} = \frac{3-0}{2} = 1.5$

$c = \frac{max + min}{2} = 1.5$

$d = 0$ w/ negative sine.

$P = 180 - (-60) = 240°. \Rightarrow k = \frac{360}{P} = \frac{360}{240} = \frac{3}{2}$

General eqn: $f(\theta) = a \frac{\sin}{\cos}(k(\theta - d)) + c \quad || \quad f(\theta) = -1.5\sin\left(\frac{3}{2}(\theta)\right) + 1.5$

22. The 8th term of an arithmetic sequence is 3 and the 100th term is 49. What is the recursive formula for the sequence?

Recall: A recursive formula requires 2 things: ① A starting place, and ② a way to "get to the next term"

Recursive Formula.
$t_1 = a$
$t_n = t_{n-1} + d$
$\Rightarrow$ need a & d

$t_{100} - t_8 = (92)d$
$\Rightarrow 49 - 3 = 92d$
$\Rightarrow d = \frac{46}{92} = \frac{1}{2}$

$t_{100} = a + 99d$
$\Rightarrow 49 = a + 99\left(\frac{1}{2}\right)$
$\Rightarrow a = 49 - 49.5 = -0.5 = -\frac{1}{2}$

$\therefore$ Recursive Formula $t_1 = -\frac{1}{2}$
$t_n = t_{n-1} + \frac{1}{2}$

23. Determine the number of terms in the sequence: –45, –32, –19, –6, ..., 124.

+13 +13 +13

t_1 t_2 t_n

$t_1 = -45$
$d = +13$
$t_n = 124$

$t_n = a + (n-1)d$
$\Rightarrow 124 = -45 + (n-1)(13)$
$\Rightarrow 169 = (n-1)(13)$
$\Rightarrow n - 1 = 13$
$\Rightarrow n = 14$

$\therefore$ 14 terms in the sequence.

24. The 3rd term of a geometric sequence is 36, and the 6th term is $\frac{9}{2}$. What is the recursive formula for the sequence?

$\frac{t_6}{t_3} = \frac{ar^5}{ar^2} = r^3$

info to use

$\frac{t_6}{t_3} = r^3$

$\Rightarrow r^3 = \frac{\frac{9}{2}}{36} = \frac{9}{72} = \frac{1}{8}$

$\therefore r^3 = \frac{1}{8}$
$\Rightarrow r = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{2}$

need t_1 & r

$t_3 = ar^2$
$\Rightarrow 36 = a\left(\frac{1}{2}\right)^2$
$\Rightarrow 36 = \frac{1}{4}a \Rightarrow a = 144$

recursive formula
$t_1 = 144$
$t_n = \left(\frac{1}{2}\right)t_{n-1}$

25. Calculate the sum of the series: –396 – 308 – 220 – 132 – ... + 836.

$t_1 = a = -396$

$t_n = 836$

$d = t_2 - t_1$

$= -308 - (-396)$

$= 88$

$S_n = \frac{n(t_1 + t_n)}{2}$

need n

$\Rightarrow S_{15} = \frac{15(-396 + 836)}{2}$

$= 3300$

$t_n = a + (n-1)d$

$836 = -396 + (n-1)(88)$

$\Rightarrow 1232 = 88(n-1)$

$\Rightarrow n - 1 = 14$

$\Rightarrow n = 15$

26. A farmer has to plant seeds in a triangular field. He knows that the middle row, row 26, needs 4025 seeds, and that the last row needs 7525 seeds. If the number of seeds planted in each row follows an arithmetic series, how many total seeds does he need?

middle

$\underbrace{1, 2, 3, \ldots}_{25}, 26, \underbrace{\ldots, n}_{25}$

$n = 26 + 25$

$= 51$

$\Rightarrow t_{51} = 7525$

$t_{51} - t_{26} = 25d$

$\Rightarrow 7525 - 4025 = 25d$

$\Rightarrow 3500 = 25d$

$\Rightarrow d = 140$

$t_{26} = a + 25d$

$\Rightarrow 4025 = a + 25(140)$

$\Rightarrow a = 525$

we have 1st and last term

$S_n = \frac{n(t_1 + t_n)}{2}$

$\Rightarrow S_{51} = \frac{51(525 + 7525)}{2}$

$= 205\,275$

27. Calculate the sum of the Geometric series $8 - 4 + 2 - 1 + \ldots + \frac{1}{8}$.

t_1 t_n

$a = 8$

$r = \frac{-4}{8} = -\frac{1}{2}$

$t_{n+1} = t_n(r)$

$= \frac{1}{8}\left(-\frac{1}{2}\right) = -\frac{1}{16}$

$S_n = \frac{t_{n+1} - t_1}{r - 1} = \frac{-\frac{1}{16} - 8}{-\frac{1}{2} - 1} = \frac{-\frac{129}{16}}{-\frac{3}{2}} = \frac{-129}{16} \times -\frac{2}{3}$

$\frac{128}{16}$

$= \frac{43}{8} = 5.375$

28. The sum of a geometric sequence $2 - 6 + 18 - 54 + \ldots - t_n = -29\,524$. Find the number of terms.

$t_1 = 2$

$r = \frac{t_2}{t_1} = \frac{-6}{2} = -3$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$\Rightarrow -29\,524 = \frac{2((-3)^n - 1)}{-4}$

$\Rightarrow 118\,096 = 2((-3)^n - 1)$

$\Rightarrow 59\,048 = (-3)^n - 1$

$\Rightarrow (-3)^n = 59049$

$\Rightarrow (-3)^n = (-3)^{10}$

$\Rightarrow \underline{n = 10}$

29. What is the total amount owed after 1, 2, and 3 years on a loan of \$3500 at 6.8%/a simple interest?

Given	Want
$P = 3500$	$A(1)$
$r = 0.068$	$A(2)$
	$A(3)$

$A(t) = P(1 + rt)$

$A(1) = 3500(1 + 0.068(1)) = 3738$

$A(2) = 3500(1 + 0.068(2)) = 3976$

$A(3) = 3500(1 + 0.068(3)) = 4214$

30. Renata invests \$10 000 in an account that earns 4.5%/a compounded quarterly for 25 years What will be the value of her investment after 25 years?

Given	Want
$P = 10\,000$	A
$i = \frac{0.045}{4}$	
$n = (4)(25) = 100$	

$A = P(1 + i)^n$

$= 10\,000\left(1 + \frac{0.045}{4}\right)^{100}$

$= \$30\,609.30$

31. Enzo buys a new car and at an interest rate of 7.8% compounded monthly. Five years later, he paid a total of $28 469.78 for the principal and interest. How much did the car originally cost?

Given	Want
$A = 28469.78$	P
$i = \frac{0.078}{12} = 0.0065$	
$n = (12)(5) = 60$	

$$P = \frac{A}{(1+i)^n} = \frac{28469.78}{(1.0065)^{60}}$$

$$= \$19{,}300$$

32. Bruce decides to save $1200 every month at 2.4%/a compounded monthly for 40 years. What is the value of Bruce's savings account at the end of 40 years?

Given	Want
$R = 1200$	F.V.
$i = \frac{0.024}{12} = 0.002$	
$n = (12)(40) = 480$	

$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{1200\left((1.002)^{480} - 1\right)}{0.002}$$

$$= \$965{,}516.27$$

(Note Bruce paid $(1200)(480) = 576\ 000$ ⇒ almost $400 000 FREE MONEY!)

33. How much do you need to invest in an account today so that you can be *regularly* paid $2000 per month for 25 years if the account pays 3%/a? How much interest do you earn over the 25 years?

Given	Want
$R = 2000$	P.V.
$i = \frac{0.03}{12} = 0.0025$	
$n = (12)(25) = 300$	

$$P.V. = \frac{R\left(1 - (1+i)^{-n}\right)}{i}$$

$$= \frac{2000\left(1 - (1.0025)^{-300}\right)}{0.0025}$$

$$= \$421\ 752.91$$

Note: $2000 per month for 25 years is a total of $600,000 ⇒ almost $180 000 FREE MONEY!)

Study well – blessings