

# Functions 11

## Course Notes

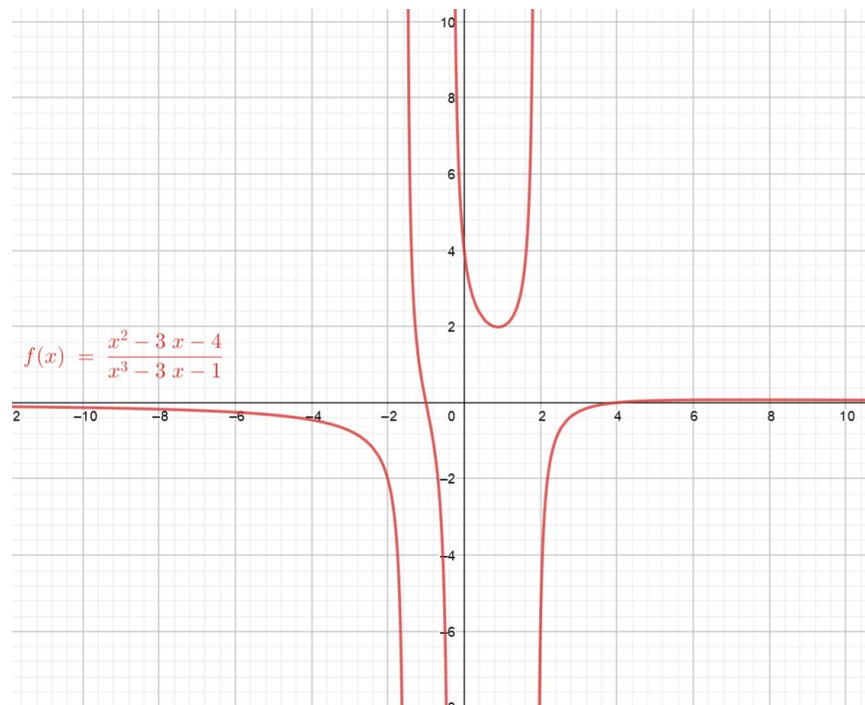
# Unit 1 – Rational Expressions

### **FACTORED FORM SAVES THE UNIVERSE**

*(developed from Chapter 2 of your text)*

We will learn

- *how to simplify polynomials through expanding and factoring*
- *how to simplify rational expressions involving the four basic operations of arithmetic*
- *how to determine restrictions on the domain for rational expressions*
- *how to determine whether two expressions are equivalent*





# Unit 1 – Polynomial and Rational Expressions

## 2.1: Adding and Subtracting Polynomials (so easy it's ridiculous)

**Learning Goal:** We are learning how to add (subtract) polynomial expressions

### Definition 2.1.1

A **SINGLE VARIABLE POLYNOMIAL** is a mathematical expression constructed by combining (through addition/subtraction) **POWER FUNCTIONS**. The polynomial is (usually) written in decreasing **order (degree)** of the power functions (and no “power” has more than one “term”). **MULTIVARIABLE POLYNOMIALS** also exist, but for **FUNCTIONS 11 we will only consider polynomials with a single variable.**

### Definition 2.1.2

A **POWER FUNCTION** is of the form  $y = c \cdot x^n$  where “ $n$ ” is the power, and “ $c$ ” is a **real number** called the **coefficient**.

e.g. of power Functions (with their orders):

e.g.'s of Polynomial Expressions (with their orders):

single variable

multivariable

One word that requires a closer look: **TERM**

### Definition 2.1.3

In any expression a **TERM** is **CONSTRUCTED BY MULTIPLYING FACTORS TOGETHER**. Terms are separated from each other by addition and subtraction. Polynomials contain “many terms” (in fact “polynomial” literally means “many terms”.) **THE TERMS OF A POLYNOMIAL EXPRESSION ARE ALL POWER FUNCTIONS. (We will look at this again!)**

Two power functions are called “**like terms**” if they have the **same order/degree/power** (but they certainly can have different coefficients!).

e.g.:

When simplifying and adding/subtracting polynomial expressions we combine like terms by adding/subtracting the coefficients of the like terms.

### Example 2.1.1

a) Simplify:  $(4x^2 - 5x^3 - 5) + (2x^3 - 4x + 2)$

b) Subtract  $4x^2 - 3x + 1$  from  $6x^2 + x$

**Final Note:** Polynomial Expressions (or functions too) are considered **equivalent** if they contain **exactly the same terms**. So, you can tell if two expressions are equivalent just by looking at them to see if they contain the same terms...OR we can tell that two mathematical objects are equivalent if they have no difference (eg. The mathematical objects  $a$  and  $b$  are equivalent if  $a - b = 0$ )

#### Success Criteria:

- I can determine the sum (difference) of two polynomials by adding (subtracting) their coefficients on terms with like orders
- I can determine whether two polynomial expressions are equivalent by graphing, or by simplifying them into the same expression.

#### Class/Homework (Section 2.1 in text)

- 1) Read Example 3 on page 87 for another view on **equivalence**
- 2) Pg 88 – 89 #4bdf, 5 – 7, 10 – 11
- 3) For the adventurous: Pg. 90 # 15, 16 (especially 16!)

# Unit 1 – Polynomial and Rational Expressions

## 2.2, 2.3: Multiplying and Factoring Polynomials

**Learning Goal:** We are learning how to multiply and factor polynomial expressions

### **2.2: MULTIPLYING REQUIRES *THE DISTRIBUTIVE PROPERTY***

Here we see two “new” concepts (in addition to what you saw last day):

1) **The Distributive Property** (for multiplication over addition)

2) **The Associative Property** (for multiplication)

Note: you may also see the **Commutative Property**, but it's not so important to today's stuff

#### Example 2.2.1:

Expand and Simplify:

a)  $3x(2x^2 - 3x + 2)$

b)  $(2x - 5)(3x + 1)$

**Double Distributive**

c)  $(5 - 2x)^2$

See Example 3, Pg. 93  
about **equivalence**

d)  $(x + 2)(3x - 1)(x + 5)$

Associate!!

e)  $(x - 2y + 3z)^2$

## 2.3: FACTORING UNDOES THE DISTRIBUTIVE PROPERTY

On the class video page (linked in Edsby) you will find videos demonstrating the skills necessary for factoring. Watch those videos and make notes for yourself. I will look at your notes. Next day we will have a full period to practice the skills seen in the videos.

### Success Criteria:

- I can determine the product of two polynomials by extending the distributive property. Sometime we call this "FOIL"
- I can invert expanding by FACTORING a polynomial expression. I can use the techniques: Common Factoring; Decomposition; Difference of Squares; Perfect Square Trinomials; Grouping.

### Class/Homework: Section 2.2

- 1) Read Examples 2 and 3 on Pgs 92 – 93 (*In Ex 2, do you prefer Fred's or Atish's solution? Why? In Ex 3 which solution do you prefer? Why? Are both entirely reliable?*)
- 2) Pg 95 #4ace, 5, 6ace, 7 (Hint: *Lee > Mathias... WHY do I make that claim?*), 9, 11

### Section 2.3

- 1) Watch the factoring videos
- 2) Problems sets will be distributed next day.

# Unit 1 – Polynomial and Rational Expressions

## 2.4: Simplifying Rational Expressions

**Learning Goal:** We are learning the definition of a Rational Expression and how to simplify them

A **RATIONAL EXPRESSION** is constructed by “**DIVIDING**” one **POLYNOMIAL EXPRESSION** by another.

e.g.  $\frac{3x^2 - 4x + 1}{x^2 - 1}$  is a rational expression, but  $\frac{3\sqrt{x} - 4}{x^2}$  is not.

**SIMPLIFYING RATIONAL EXPRESSIONS REQUIRES 4 THINGS:**

- 1) Factoring any polynomials which can be factored
- 2) **Stating any restrictions on the variables**
- 3) Cancelling any common factors, top to bottom
- 4) Writing the rational expression in simplified form

Note that stating restrictions **MUST BE DONE BEFORE CANCELLING!!!!** If you cancel before stating the restrictions, **YOU RUN THE RISK OF EXPLODING THE UNIVERSE**. Don't do it. **FOR THE SAKE OF ALL HUMANITY, PLEASE DON'T DO CANCEL BEFORE STATING RESTRICTIONS!**

**RESTRICTIONS ON A RATIONAL EXPRESSION**

Consider the rational expression  $\frac{2x-5}{x+2}$ . Because  $x$  is a variable, we can substitute different (varying) values for it and calculate different values for the rational expression.

However, there is one value which we cannot substitute:

**Example 2.4.1**

Simplify, stating any restrictions on the variable:  $\frac{3a(a-2b)}{(a+b)(a-2b)}$

**Example 2.4.2**

Simplify  $\frac{x^2-9}{(2+x)(3-x)}$

**Example 2.4.3**

Simplify, stating any restrictions on the variable:  $\frac{3t^2+t-2}{9t^3-6t^2}$ .

**Success Criteria:**

- I can determine whether an expression is rational or not
- I can simplify rational expressions by factoring and cancelling common factors.
- I can state restriction for rational expressions by not allowing the denominator to equal zero.

**Class/Homework**

Pg. 112 – 114 #2, 3bc, 4acf, 5, 8a, 10, 11

## Unit 1 – Polynomial and Rational Expressions

### 2.6: Multiplying and Dividing Rational Expressions

**Learning Goal:** We are learning how to multiply and divide rational expressions

This concept extends what we learned in section **2.4: Simplifying Rational Expressions.**, BUT we are adding a small (but **ridiculously fun**) twist.

Now we will consider more than one rational expression at a time, and **MULTIPLY OR DIVIDE** them. Since Rational Expressions are analogous to fractions, we need to remind ourselves of the rules for multiplying and dividing fractions.

#### MULTIPLYING FRACTIONS

**The Rule is:**

#### DIVIDING FRACTIONS

**The Rule is:**

When **MULTIPLYING RATIONAL EXPRESSIONS** we need to do the following (familiar) things:

- 1) Factoring any polynomials which can be factored
- 2) **Stating any restrictions on the variable(s)**
- 3) Cancelling any common factors, top to bottom (even from one expression to the other)
- 4) Writing the rational expression in simplified form

When **DIVIDING RATIONAL EXPRESSIONS**, we do the same four things, **BUT** when stating the restrictions, there is one small twist **BECAUSE WE ARE NOT ALLOWED TO DIVIDE BY ZERO**. (Sorry, I shouldn't be shouting, but this is important. It would be terrible if you divided by zero, and then were taken away by the math-cops for endangering the universe).

### Examples

a)  $\frac{3(x-2)}{2x} \times \frac{6}{x-2}$

Why have I coloured the denominators?

b)  $\frac{x^2-25}{3x^2+x-2} \times \frac{6x^2-13x+6}{2x^2+7x-15}$

c)  $\frac{(2x+1)^2}{25x^2-1} \div \frac{-4x-2}{20x+4}$

d)  $\frac{9y^2-4}{4y-12} \div \frac{9y^2+12y+4}{18-6y}$

**Success Criteria:**

- I can multiply rational expressions by: Factoring the numerators and denominators; Stating restrictions by finding zeros of the denominator; Cancelling common factors (top to bottom); then finally writing the expression in simplified form.
- I can divide rational expressions by: Factoring the numerators and denominators; Stating restrictions by finding zeros of the denominator; Multiplying by the **reciprocal** of the **divisor**; Cancelling common factors (top to bottom); then finally writing the expression in simplified form.

**Class/Homework**

Pg. 121 – 123 #3, 5, 6abc, 8 – 11

## Unit 1 – Polynomial and Rational Expressions

### 2.7: Adding and Subtracting Rational Expressions

**Learning Goal:** We are learning how to add (subtract) rational expressions

**This is it.** The pinnacle of Rational Expressions. **The most difficult thing you can do with Rational Expressions is to add or subtract them.** That's right! **Adding and subtracting is the most difficult thing.** Thankfully, you all can handle it!!

# COMMON DENOMINATOR

Getting a Common Denominator is the key to the whole scene. A “**COMMON**” denominator must contain enough **FACTORS** (*or better: THE CORRECT FACTORS*) to “**KEEP EVERYONE HAPPY**”, so to speak. Keep in mind that we **STILL MUST STATE RESTRICTIONS** when faced with factors containing variables!!!

In order to simply Rational Expressions being added or subtracted, you must:

- 1) Factoring any polynomials which can be factored
- 2) **Stating any restrictions on the variable(s)**
- 3) Determine the **COMMON DENOMINATOR** by identifying all **needed** factors
- 4) Writing the rational expressions with the **needed** factors so that each expression has the same denominator.
- 5) Add/Subtract the numerators of the expressions (*which may require some expanding so that you can collect like terms!*)
- 6) Check to see if the simplified numerator can be factored, and if it can, factor it.
- 7) Write the sum/difference in simplified (factored) form

#### Example 2.7.1

Simplify, stating any restrictions.

$$\frac{3}{t^4} + \frac{1}{2t^2} - \frac{3}{5t}$$

**Example 2.7.2**

Simplify and state any restrictions.

$$\frac{4x}{x^2 + 6x + 8} - \frac{3x}{x^2 - 3x - 10}$$

**Example 2.7.3**

Simplify. State restrictions. BEDMAS, folks, BEDMAS!!!!!!!!!!!! (Seriously...bedmas)

$$\frac{p+1}{p^2 + 2p - 35} + \frac{p^2 + p - 12}{p^2 - 2p - 24} \times \frac{p^2 - 4p - 12}{p^2 + 2p - 15}$$

**Success Criteria:**

- I can determine the sum (difference) of two rational expressions by: determining the **LCD** (lowest common denominator) of the denominators; stating any restrictions for the expressions; rewriting each expression using that common denominator; combine the expressions into a single expression; simplifying the numerator by collecting like terms.

**Class/Homework:** Pg 128 – 130 #3bc, 5cd, 6abef, 7acef, 8, 9abd