

Unit 1 – Polynomial and Rational Expressions

2.7: Adding and Subtracting Rational Expressions

Learning Goal: We are learning how to add (subtract) rational expressions

This is it. The pinnacle of Rational Expressions. **The most difficult thing you can do with Rational Expressions is to add or subtract them.** That's right! **Adding and subtracting is the most difficult thing.** Thankfully, you all can handle it!!

COMMON DENOMINATOR

Getting a Common Denominator is the key to the whole scene. A “**COMMON**” denominator must contain enough **FACTORS** (*or better: THE CORRECT FACTORS*) to “**KEEP EVERYONE HAPPY**”, so to speak. Keep in mind that we **STILL MUST STATE RESTRICTIONS** when faced with factors containing variables!!!

In order to simply Rational Expressions being added or subtracted, you must:

- 1) Factoring any polynomials which can be factored
- 2) **Stating any restrictions on the variable(s)**
- 3) Determine the **COMMON DENOMINATOR** by identifying all **needed** factors
- 4) Writing the rational expressions with the **needed** factors so that each expression has the same denominator.
- 5) Add/Subtract the numerators of the expressions (*which may require some expanding so that you can collect like terms!*)
- 6) Check to see if the simplified numerator can be factored, and if it can, factor it.
- 7) Write the sum/difference in simplified (factored) form

all "unique" factors
↓

Example 2.7.1

Simplify, stating any restrictions.

$$\frac{3}{t^4} + \frac{1}{2t^2} - \frac{3}{5t} \quad \text{restrictions: } t \neq 0 \quad \text{CD } (t^4)(2)(5) = 10t^4$$

$$= \frac{3}{t^4} \cdot \frac{10}{10} + \frac{1}{2t^2} \cdot \frac{5t^2}{5t^2} - \frac{3}{5t} \cdot \frac{2t^3}{2t^3}$$

$$= \frac{30}{10t^4} + \frac{5t^2}{10t^4} - \frac{6t^3}{10t^4}$$

$$= \frac{30 + 5t^2 - 6t^3}{10t^4}$$

Example 2.7.2

Simplify and state any restrictions.

$$\frac{4x}{x^2+6x+8} - \frac{3x}{x^2-3x-10}$$

$$= \frac{4x}{(x+4)(x+2)} - \frac{3x}{(x-5)(x+2)}$$

restrictions
 $x \neq -4, -2, 5$

$$\begin{array}{r} x^2+6x+8 \\ \hline 4, 2 \\ \hline \end{array}$$

$$\begin{array}{r} x^2-3x-10 \\ \hline x \quad + \\ -10 \quad -3 \\ \hline \end{array}$$

$(x-5)(x+2) \leftarrow -5, 2$

CD: $(x+4)(x+2)(x-5)$

$$= \frac{4x(x-5)}{(x+4)(x+2)(x-5)} - \frac{3x(x+4)}{(x+4)(x+2)(x-5)} = \frac{4x(x-5) - 3x(x+4)}{(x+4)(x+2)(x-5)}$$

$$= \frac{4x^2 - 20x - 3x^2 - 12x}{(x+4)(x+2)(x-5)} = \frac{x^2 - 32x}{(x+4)(x+2)(x-5)} = \frac{x(x-32)}{(x+4)(x+2)(x-5)}$$

Example 2.7.3

Simplify. State restrictions. BEDMAS, folks, **BEDMAS**!!!!!!!!!!!! (Seriously...bedmas)

$$\frac{p+1}{p^2+2p-35} + \frac{p^2+p-12}{p^2-2p-24} \times \frac{p-6}{p^2+2p-15}$$

$$= \frac{p+1}{(p+7)(p-5)} + \frac{(p+4)(p-3)}{(p-6)(p+4)} \times \frac{p-6}{(p-3)(p+5)}$$

restrictions $p \neq 7, 5, 6, -4, 3, -5$

$$= \frac{p+1}{(p+7)(p-5)} + \frac{1}{p+5}$$

CD: $(p+7)(p-5)(p+5)$

Success Criteria:

- I can determine the sum (difference) of two rational expressions by: determining the LCD (lowest common denominator) of the denominators; stating any restrictions for the expressions; rewriting each expression using that common denominator; combine the expressions into a single expression; simplifying the numerator by collecting like terms.

$$= \frac{p+1}{(p+7)(p-5)} + \frac{1}{p+5} \quad \text{CD } (p+7)(p-5)(p+5)$$

$$= \frac{p+1}{(p+7)(p-5)} \cdot \frac{(p+5)}{(p+5)} + \frac{1}{p+5} \cdot \frac{(p+7)(p-5)}{(p+7)(p-5)}$$

$$= \frac{(p+1)(p+5) + (1)(p+7)(p-5)}{(p+7)(p+5)(p-5)}$$

$$= \frac{p^2 + 5p + p + 5 + p^2 - 5p + 7p - 35}{(p+7)(p+5)(p-5)}$$

$$= \frac{2p^2 + 8p - 30}{(p+7)(p+5)(p-5)} = \frac{2(p^2 + 4p - 15)}{(p+7)(p+5)(p-5)}$$