Chapter 1 – Introduction to Functions

1.1 Relations and Functions (*This is a KEY lesson*!)

Learning Goal: We are learning to recognize functions in various representations.

This course is called **FUNCTIONS**, so it seems rather important that you know what a function actually is. Thus you need to know, very well, the following (algebraic) definition:

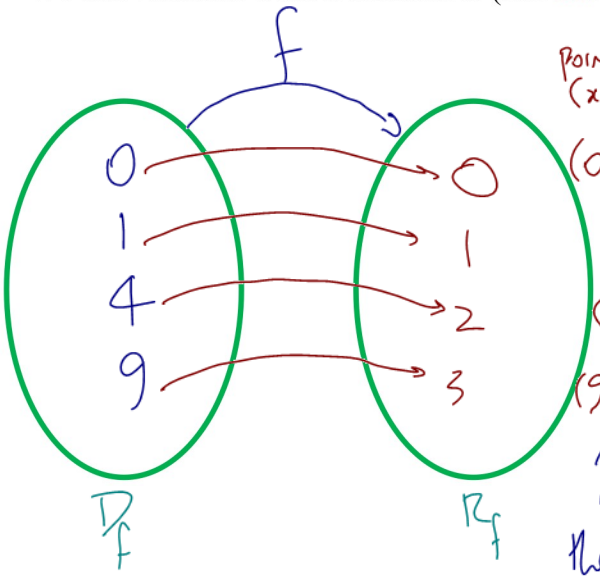
Definition 1.1.1

A **FUNCTION** is an algebraic rule (eg $3x^2-2x+1$) which assigns (connects) exactly one value in a set called the range to each value in a set called the domain

fn = function

We can visualize what a function is (and **isn't**) by using so-called "**arrow diagrams**":

f is a fn

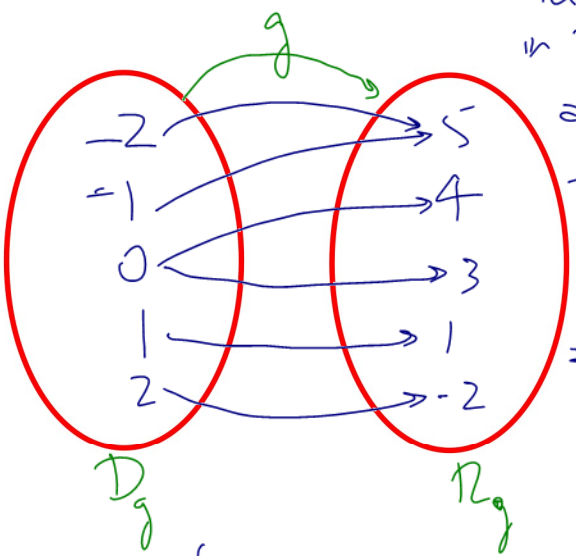


POINTS (x,y)
(0,0)
(1,1)
(4,2)
(9,3)

the "rule" f makes points by connecting domain and range "x" "y"

D_f: {0, 1, 4, 9}

R_f: {0, 1, 2, 3}



the # 0 in D_g is assigned to two range values (4 & 3) ⇒ g is NOT a fn

D_g: {-2, -1, 0, 1, 2}

R_g: {5, 4, 3, 1, -2}

It appears the RELATIONSHIP between "x" & "y" (by f) is square-rooting

We need a few more definitions before moving on, so that we can "speak the language" of functions (and that language is mathematics!)

Definition 1.1.2

A **SET** is a collection of things. In math sets are collections of numbers. The NOTATION we use to "say" set is "{ }" (braces)

Definition 1.1.3

A **RELATION** is a relationship (algebraic) between 2 sets of numbers (domain & range). Some relations are "functional" – we call those FUNCTIONS. Some relationships are not functions – we just call those "relations"

Definition 1.1.4

The **DOMAIN** of a function (or a relation) is the set of "input values" (often the variable "x" is used for domain)

Definition 1.1.5

The **RANGE** of a function (or a relation) is the "calculated" "output value". (plug a number in – the "rule" does its stuff, and out pops another number)

Two other important terms to know are:

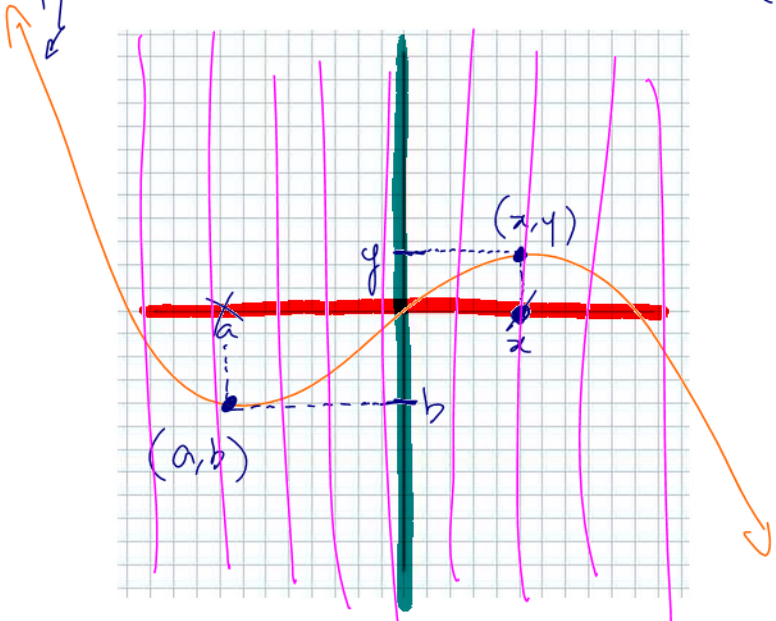
1) The **INDEPENDENT VARIABLE** is the "domain variable" (usually 'x', often "t")

2) The **DEPENDENT VARIABLE** is the 'range variable' (denoted "f(x)" – old school "y")

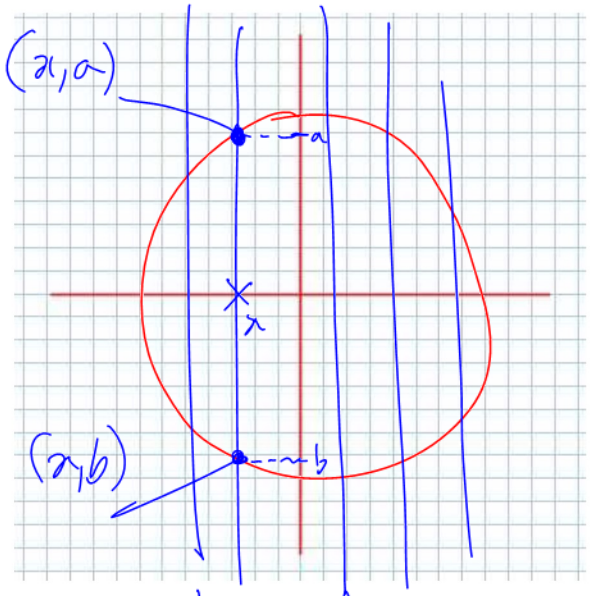
If vertical lines cut a 'graph' in only one place (for every vertical line) then the relation is a fn. If vertical lines cut a curve in more than one place it is not a fn (just a relation)

KNOWING WHEN A RELATION IS, AND ISN'T, A FUNCTION

Graphically: *The Vertical Line Test* (VLT)



f is a function - it passes the VLT



one domain value is assigned to two range values

Algebraically: (*NOTE: this is a "rough" way of thinking about the problem*) (fails the VLT)

If the Dependent Variable ("y") (range variable) is raised to an even power, the algebraic rule is NOT a fn.

e.g. $y = x^2 - 5$ is a function. $y^3 = x - 7$ is a fn

$y^2 = x - 2$ is NOT a fn

Success Criteria:

- I can determine the domain of a relation or function as the set of all values of the independent variable
- I can determine the range of a relation or function as the set of all values of the dependent variable
- I can apply the vertical line test to determine if a graph is a function
- I can recognize whether a relation is a function from its equation