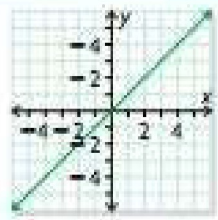
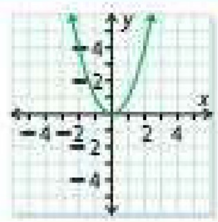
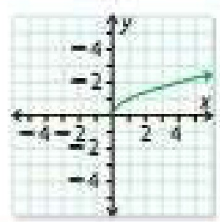
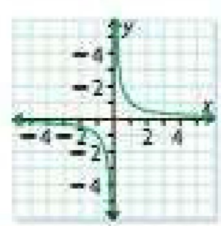
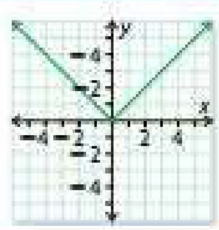
# Chapter 1 – Introduction to Functions

## 1.3 and 1.4 Parent Functions and Domain and Range

**Learning Goal:** We are learning the graphs and equations of five basic functions; and using their tables, graphs, or equations to find their domains and ranges.

We will be closely studying **5 TYPES OF FUNCTIONS** (***Actually we'll study more than the following five, but for now….the big five are***: )

| Equation of Function | Name of Function | Sketch of Graph |
|---|---|---|
| $f(x) = x$ | linear function |  |
| $f(x) = x^2$ | quadratic function |  |
| $f(x) = \sqrt{x}$ | square root function |  |
| $f(x) = \frac{1}{x}$ | reciprocal function |  |
| $f(x) = \lvert x \rvert$ | absolute value function |  |

# DOMAIN AND RANGE

$x$-values ($x$-axis) (horizontal axis)

functional-values ($y$-values) functional axis (vertical axis)

Two **INCREDIBLY IMPORTANT** aspects of functions are their domain and range

read left-to-right

Again, the Domain is the SET of input values (independent value)

And, the Range is the SET of output value (dependent values)

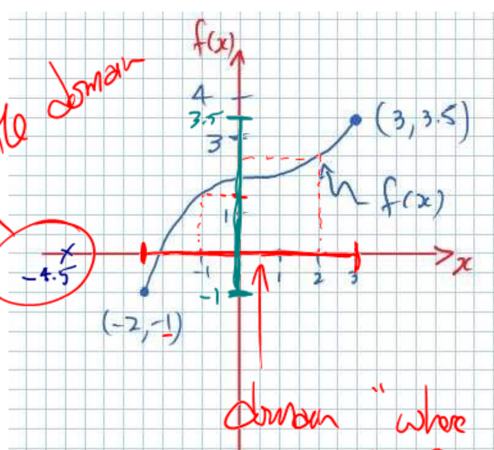
read bottom-to-top

**Example 1.4.1**

Given the **SKETCH OF THE GRAPH** of the **RELATION** determine: the domain, the range of the relation, and whether the relation is, or is not, a function.

SET builder notation!

a)




not in the domain

domain "where the fn has inputs"

"set of real #'s"

"belongs to"

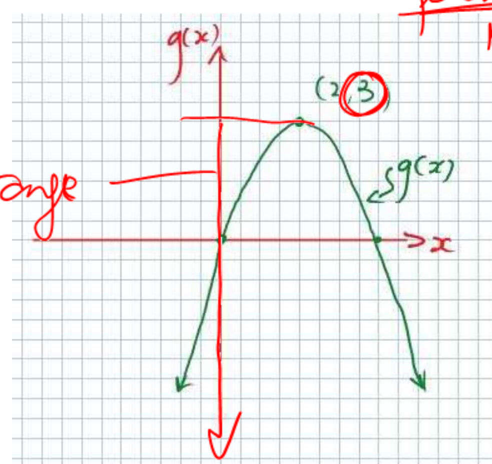
there is a restriction

$D_f = \{x \in \mathbb{R} \mid -2 \le x \le 3\}$

interval of all $x$-values between $-2$ & $3$.

$R_f = \{f(x) \in \mathbb{R} \mid -1 \le f(x) \le 3.5\}$

b)




range

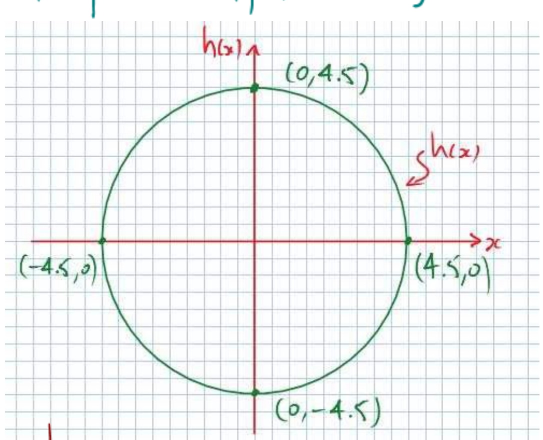
parabolas have no restriction on domain

but range is always restricted by the vertex

$D_g = \{x \in \mathbb{R}\}$

$R_g = \{g(x) \in \mathbb{R} \mid g(x) \le 3\}$

c)

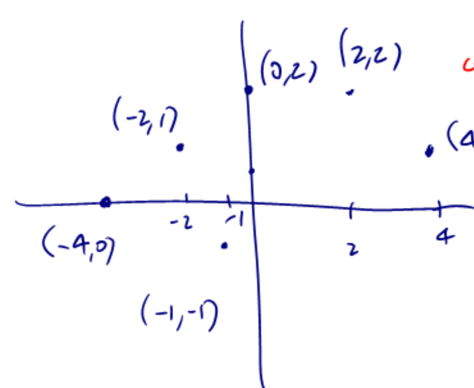



$h(x)$ is not a fn (fails the VLT)

$D_h = \{x \in \mathbb{R} \mid -4.5 \le x \le 4.5\}$

$R_h = \{h(x) \in \mathbb{R} \mid -4.5 \le h(x) \le 4.5\}$

d)




$D_p = \{-4, -2, -1, 0, 2, 4\}$

$R_p = \{0, 1, -1, 2\}$

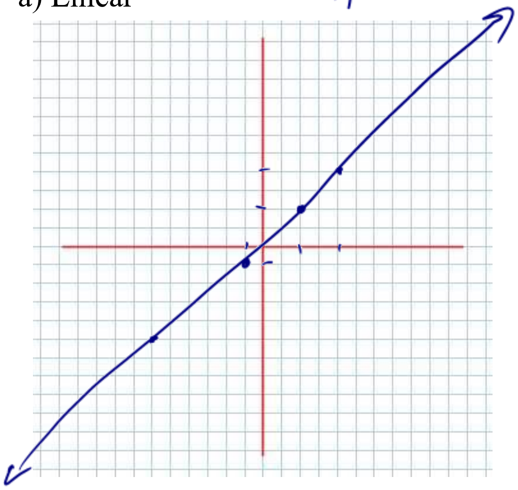
the most basic fn. (no extra #'s)

# THE PARENT FUNCTIONS *(for Grade 11)*

Together we will explore (graphically) basic properties of the five *parent* functions:

a) Linear

$f(x) = x$



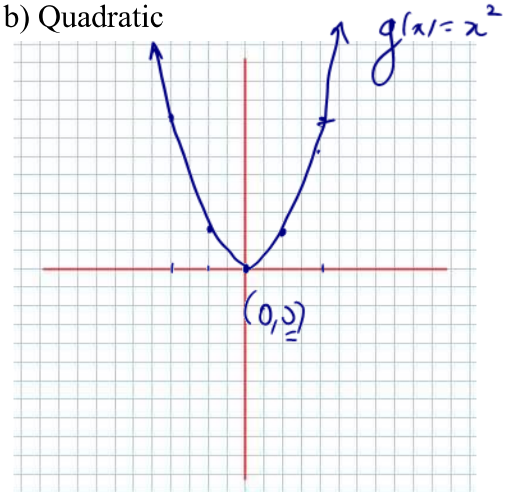
$D_f = \{x \in \mathbb{R}\}$

$R_f : \{f(x) \in \mathbb{R}\}$

True for all linear fns except
- horizontal lines
- vertical lines

b) Quadratic

$g(x) = x^2$




$D_g = \{x \in \mathbb{R}\}$

All quadratics have unrestricted domains

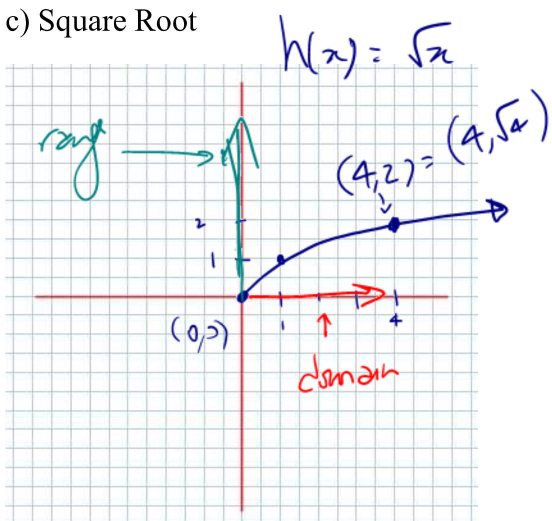
$R_g = \{g(x) \in \mathbb{R} \mid g(x) \geq 0\}$

range is restricted by vertex and the direction of opening.

c) Square Root

$h(x) = \sqrt{x}$

Domain CANNOT have any negative values




Square root fns are restricted in both domain + range.

$D_h = \{x \in \mathbb{R} \mid x \geq 0\}$

$R_h = \{h(x) \in \mathbb{R} \mid h(x) \geq 0\}$

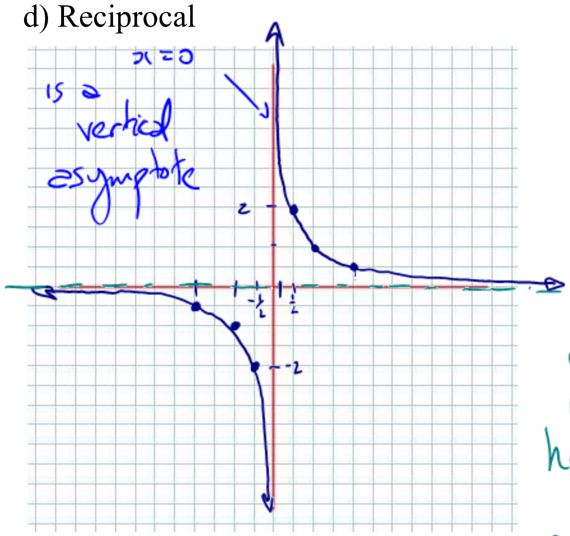
$f(x) = \frac{1}{x}$ , $x \neq 0$ domain restriction

$g(x) = |x|$

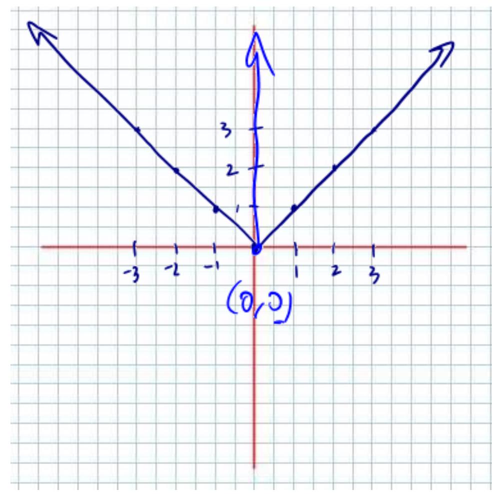
ToV.

| $x$ | $f = \frac{1}{x}$ |
|---|---|
| $-2$ | $-\frac{1}{2}$ |
| $-1$ | $-\frac{1}{1}$ |
| $-\frac{1}{2}$ | $-2$ |
| $+\frac{1}{2}$ | $2$ |
| $1$ | $1$ |
| $2$ | $\frac{1}{2}$ |

d) Reciprocal




e) Absolute Value




Def: A asymptote is a line which you cannot cross

$D_f = \{x \in \mathbb{R} \mid x \neq 0\}$

$R_f : \{f(x) \in \mathbb{R} \mid f(x) \neq 0\}$

$D_g = \{x \in \mathbb{R}\}$

$R_g = \{g(x) \in \mathbb{R} \mid g(x) \geq 0\}$

**Example 1.4.2** (*From Pg. 36 in your text*)

**8.** Write a function to describe coffee dripping into a 10-cup carafe at a rate of 2 mL/s. State the domain and range of the function (1 cup = 250 mL).

$\frac{mL}{sec}$ ← volume ← time

$2500 = 2t$
$t = \frac{2500}{2} = 1250$

Let $V(t)$ = volume of coffee in the carafe

$t$ = time in second

$V(t) = 2t$

max volume $= (250)(10) = 2500$ mL

Real world problems always have domain/range restrictions

Domain: "$t$" min time $t = 0$ max time $t = 1250$

$D : \{t \in \mathbb{R} \mid 0 \leq t \leq 1250 \text{ sec}\}$

$R : \{V(t) \in \mathbb{R} \mid 0 \leq V(t) \leq 2500 \text{ mL}\}$

restricted domain/range

$D_f : \{x \in \mathbb{R} \mid \min \leq x \leq \max\}$

**Example 1.4.3** (*From Pg. 37 in your text…use ~~Desmos~~*) GeoGebra

**9.** Determine the domain and range of each function.

**a)** $f(x) = -3x + 8$

belongs to

$D_f : \{x \in \mathbb{R}\}$

the set of all real number

$R_f : \{f(x) \in \mathbb{R}\}$

**d)** $p(x) = \frac{2}{3}(x - 2)^2 - 5$

$D_p : \{x \in \mathbb{R}\}$

$R_p : \{p(x) \in \mathbb{R} \mid p(x) \geq -5\}$

**f)** $r(x) = \sqrt{5 - x}$

$D_r = \{x \in \mathbb{R} \mid x \leq 5\}$

$R_r = \{r(x) \in \mathbb{R} \mid r(x) \geq 0\}$

Quadratics

STANDARD FORM
$f(x) = ax^2 + bx + c$

zeros
$f(x) = a(x-r)(x-s)$

vertex
$f(x) = a(x-h)^2 + k$

Note: real world problems always have restrictions on domain & range

### Example 1.4.4

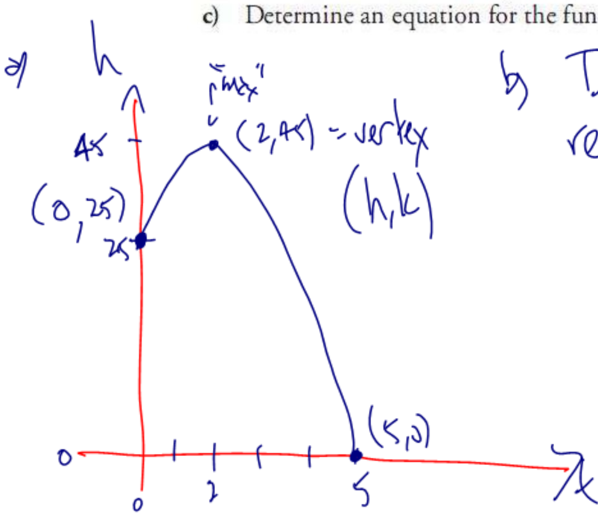
**10.** A ball is thrown upward from the roof of a 25 m building. The ball reaches a height of 45 m above the ground after 2 s and hits the ground 5 s after being thrown. max

a) Sketch a graph that shows the height of the ball as a function of time. $h(t) =$ "rule"

b) State the domain and range of the function.

c) Determine an equation for the function.

a)




b) Domain & Range must be restricted to match real world "restrictions" (eg no negative time)

$$D_h : \{t \in \mathbb{R} \mid 0 \le t \le 5\}$$

$$R_h : \{h(t) \in \mathbb{R} \mid 0 \le h(t) \le 45\}$$

c) Using vertex form (vertex $(2,45) = (h,k)$)

$$h(t) = a(t-h)^2 + k$$

$$\Rightarrow h(t) = a(t-2)^2 + 45$$

$(t, h(t))$

use $(5,0)$ to calculate 'a' $\Rightarrow 0 = a(5-2)^2 + 45$

$$\Rightarrow -45 = a(3)^2 \Rightarrow -45 = 9a$$

$$\Rightarrow \boxed{a = -5}$$

**Success Criteria:**

- I can identify the unique characteristics of five basic types of functions
- I can identify the domain and ranges of five basic types of functions
- I can identify when there are restrictions given real-world situations

$\therefore$ eqn is $h(t) = -5(t-2)^2 + 45$