Chapter 1 – Introduction to Functions

1.6 – 1.8: Transformations of Functions *(Part 1)*

BEDMAS

Learning Goal: We are learning to apply combinations of transformations in a systematic order to sketch graphs of functions.

To **TRANSFORM** something is to CHANGE IT

TRANSFORMATIONS OF FUNCTIONS can be seen in two ways: algebraically, and graphically. We'll begin by examining transformations graphically.

But before we do, we need to remember that the **GRAPH OF A FUNCTION**, $f(x)$, is given by:

$$f(x)=\left\{\left(x, f(x)\right) \mid x \in D_f\right\}$$

So, for functions we have two things (NUMBERS!) to "transform". We can apply transformations to

1) **Domain** values (which we call **HORIZONTAL TRANSFORMATIONS**)
2) **Range** values (which we call **VERTICAL TRANSFORMATIONS**)

There are **THREE BASIC FUNCTIONAL TRANSFORMATIONS**

1) Flips (*Reflections "around" an axis*)
2) Stretches (*Dilations*)
3) Shifts (*Translations*)

So, we can have **Horizontal** flips, stretches and/or shifts, and **Vertical** flips, stretches and/or shifts. Now let's take a look at how transformations can be applied to functions.

Note: We'll (mostly) be applying transformations to our so-called "parent functions" (although applying transformations to linear functions can seem pretty silly!)

Parent fns $f(x)=x$, $f(x)=x^2$, $f(x)=\sqrt{x}$, $f(x)=|x|$, $f(x)=\frac{1}{x}$

Example 1.8.1

Consider, and make observations concerning the sketch of the graph of the parent function $f(x)=\sqrt{x}$ and the transformed function $g(x)=2\sqrt{-x-2}+1$.

vertical shift

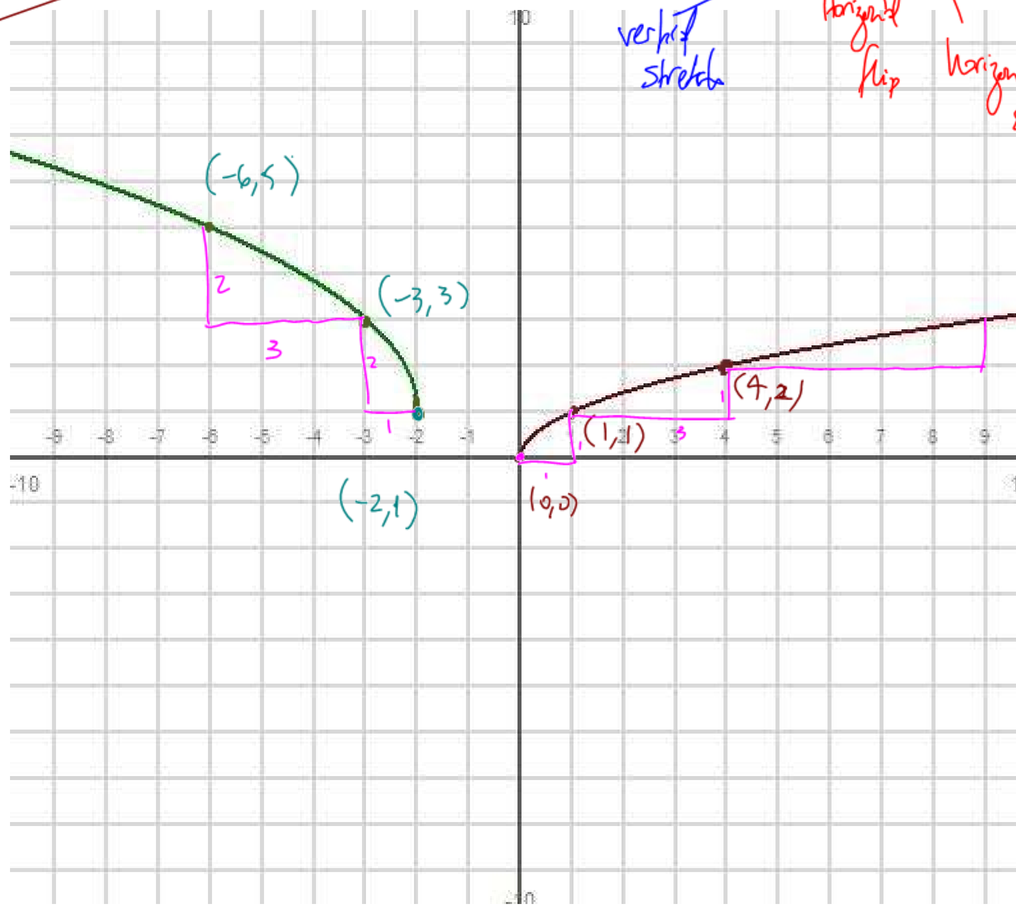
vertical stretch

horizontal flip

horizontal shift

transformed fn

parent fn (no transformations)



Horizontal Transformations

Flip: Yes!

Shift 2 units left

Vertical Transformations

Flip – No

Stretch "x2"

Shift up 1

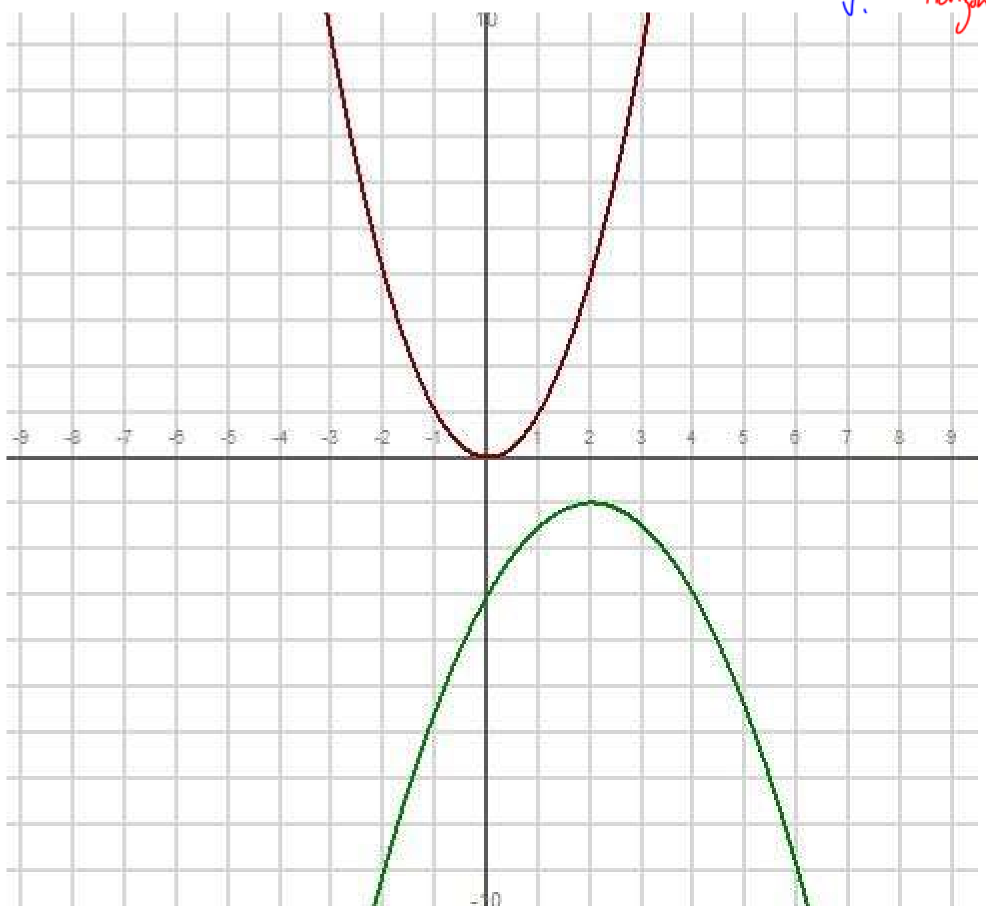
Note: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation $g(x)=2f(-x-2)+1$

recall a point $(x, f(x))$ – Anything on the "outside" of $f(x)$ is a vertical transformation. Anything on the "inside" of $f(x)$ is a horizontal transformation.

Example 1.8.2

Consider, and make observations concerning the sketch of the graph of the parent function $f(x) = x^2$ and the transformed function $g(x) = -\frac{1}{2}(1x-2)^2 - 1$

vertical   v.   Horizontal   v



	Horizontal Transformations	**Vertical Transformations**
Flip	No	Yes ("x -1")
Stretch	No (or Yes x 1)	Yes x $\frac{1}{2}$
Shift	2 units right (+2)	Yes down 1 (-1)

⇒ mean x y values by $-\frac{1}{2}$

Note: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation

$g(x) = -\frac{1}{2} f(x-2) - 1$