Chapter 1 – Introduction to Functions

1.6 – 1.8: Transformations of Functions *(Part 2)*

We now turn to examining Transformations of Functions from an algebraic point of view (although a geometric perspective will still shine though!)

→ parent fn could be $f(x)=x$ or $f(x)=x^2$ or $f(x)=\sqrt{x}$ or $f(x)=|x|$ or $f(x)=\frac{1}{x}$

Definition 1.8.1

Given a function $f(x)$ we can obtain a related function through functional transformations as STANDARD FORM "k" is factored away from x

$$g(x)=af\left(k(x-d)\right)+c$$, where

k is negative

k is the Horizontal Stretch with Stretch Factor "$\frac{1}{k}$" If $k<0$ we also have a horizontal flip } × "x" by $\frac{1}{k}$.

a is the vertical stretch If $a<0$ we also have a vertical flip } ×a

d is the horizontal shift (left or right)

c is the vertical shift (up or down)

Example 1.8.3

Consider the given function. State its parent function, and all transformations.

$$f(x)=3\sqrt{-x+2}-1$$

PARENT FN $g(x)=\sqrt{x}$

f(x) is NOT in standard form "k must be factored away from x"

STANDARD FORM $f(x)=3\sqrt{-1(x-2)}-1$

	Horizontal Transformations	**Vertical Transformations**
Flip	Yes!	No
Stretch	×1 } × −1	×3
Shift	2 units right	down 1

Example 1.8.4

outside the parent fn type

Inside the parent fn type

The basic absolute value function $f(x)=|x|$ has the following transformations applied to it: **Vertical Stretch** -3**, Vertical Shift** 1 **up, Horizontal Shift** 5 **right.** Determine the equation of the transformed function.

$$g(x) = -3|x-5|+1$$

Back to a geometric point of view

Sketching the graph of a transformed function can be relatively easy if we know:

1) The shape of the parent function AND a few (3 or 4) points on the parent.
2) How transformations affect the points on the parent
   i) Horizontal transformations affect the domain values (***OPPOSITE!!!!!!***)
   ii) Vertical transformations affect the range values

Note: Given a point on some parent function which has transformations applied to it is called an ***IMAGE POINT*** on the transformed function.

Example 1.8.5

Given the sketch of the function $f(x)$ determine the image points of the transformed function $-2f\left(\frac{1}{3}(x+1)\right)+3$ and sketch the graph of the transformed function.

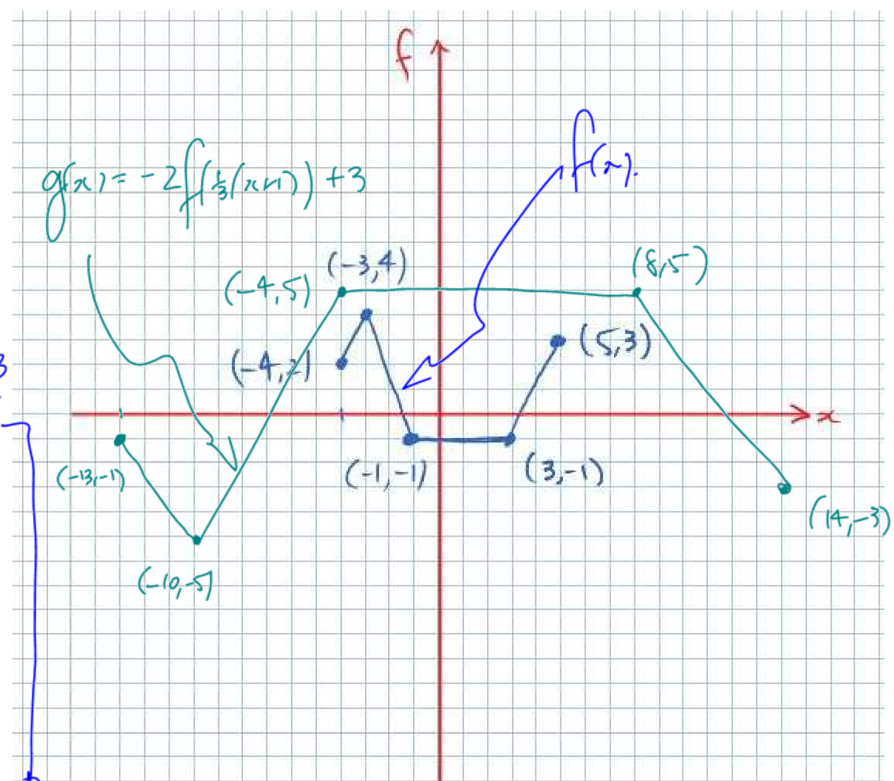
$$g(x) = \underset{V}{-2}f\left(\underset{H}{\tfrac{1}{3}}(x\underset{H}{+1})\right)\underset{V}{+3}$$

Tables of Values

parent x-value

transformed x-values

Parent		Transformed	
x_p	f	$x_T = 3x_p - 1$	$g = -2f+3$
-4	2	$3(-4)-1$ -13	$-2(2)+3$ -1
-3	4	-10	-5
-1	-1	-4	5
3	-1	8	5
5	3	14	-3



Note: the point $(-13,-1)$ (on the transformed fn) is the

IMAGE POINT of $(-4, 2)$ on the parent

STANDARD FORM (of $g(x)$)

$g(x) = 2\sqrt{-1(x-1)} - 2$

Example 1.8.6

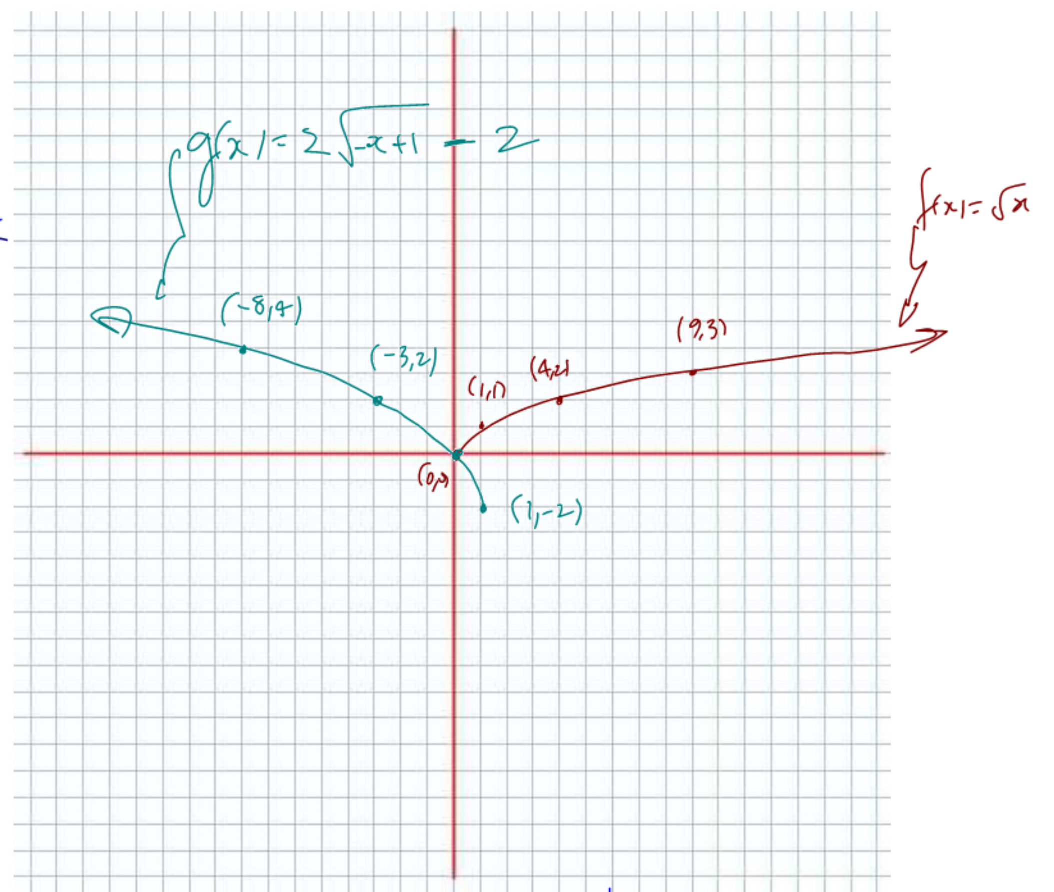
On the same set of axes sketch the graphs of $f(x)=\sqrt{x}$ and $g(x)=2\sqrt{-x+1}-2$.

factor out "k" = −1

Determine ~~three~~ four points on the parent function and state the image points for each.

Table of Values

Parent		Transformed	
x_p	$f=\sqrt{x}$	$x_T = -x_p+1$	$g = 2f-2$
0	0	1	−2
1	1	0	0
4	2	−3	2
9	3	−8	4



$D_g: \{x \in \mathbb{R} \mid x \le 1\}$

$R_g = \{g(x) \in \mathbb{R} \mid g(x) \ge -2\}$

Success Criteria:

- I can use the value of a to determine if there is a vertical stretch/reflection in the x-axis
- I can use the value of k to determine if there is a horizontal stretch/reflection in the y-axis
- I can use the value of d to determine if there is a horizontal translation
- I can use the value of c to determine if there is a vertical translation
- I can transform x coordinates by using the expression $\frac{1}{k}x + d$
- I can transform y coordinates by using the expression $ay + c$