Chapter 1 – Introduction to Functions

1.6 – 1.8: Transformations of Functions *(Part 2)*

We now turn to examining Transformations of Functions from an algebraic point of view (although a geometric perspective will still shine though!)

→ parent fn could be $f(x)=x$ OR $f(x)=x^2$ OR $f(x)=\sqrt{x}$ OR $f(x)=|x|$ OR $f(x)=\frac{1}{x}$

Definition 1.8.1

Given a function $f(x)$ we can obtain a related function through functional transformations as STANDARD FORM ("k" is factored away from x)

$$g(x)=af\left(k(x-d)\right)+c \text{, where}$$

k is the Horizontal Stretch with Stretch Factor "$\frac{1}{k}$" } × "x" by $\frac{1}{k}$. If $k<0$ (k is negative) we also have a horizontal flip

a is the vertical stretch. If $a<0$ we also have a vertical flip } × a

d is the horizontal shift (left or right)

c is the vertical shift (up or down)

Example 1.8.3

Consider the given function. State its parent function, and all transformations.

$$f(x)=3\sqrt{-x+2}-1$$

PARENT FN $g(x)=\sqrt{x}$

f(x) is NOT in standard form "k must be factored away from x"

STANDARD FORM $f(x)=3\sqrt{-1(x-2)}-1$

	Horizontal Transformations	Vertical Transformations
Flip	Yes!	No
Stretch	×1 } × −1	×3
Shift	2 units right	down 1

Example 1.8.4

The basic absolute value function $f(x) = |x|$ has the following transformations applied to it: **Vertical Stretch** -3**, Vertical Shift** 1 **up, Horizontal Shift** 5 **right.**
Determine the equation of the transformed function.

Back to a geometric point of view

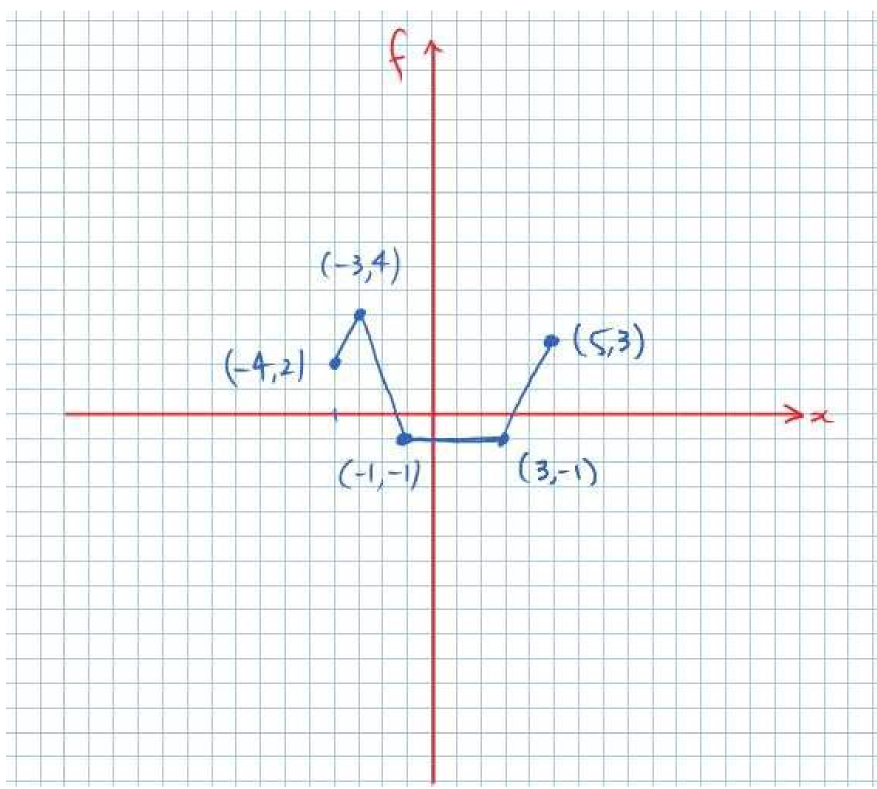
Sketching the graph of a transformed function can be relatively easy if we know:

1) The shape of the parent function AND a few (3 or 4) points on the parent.
2) How transformations affect the points on the parent
   i) Horizontal transformations affect the domain values (***OPPOSITE!!!!!!***)
   ii) Vertical transformations affect the range values

Note: Given a point on some parent function which has transformations applied to it is called an ***IMAGE POINT*** on the transformed function.

Example 1.8.5

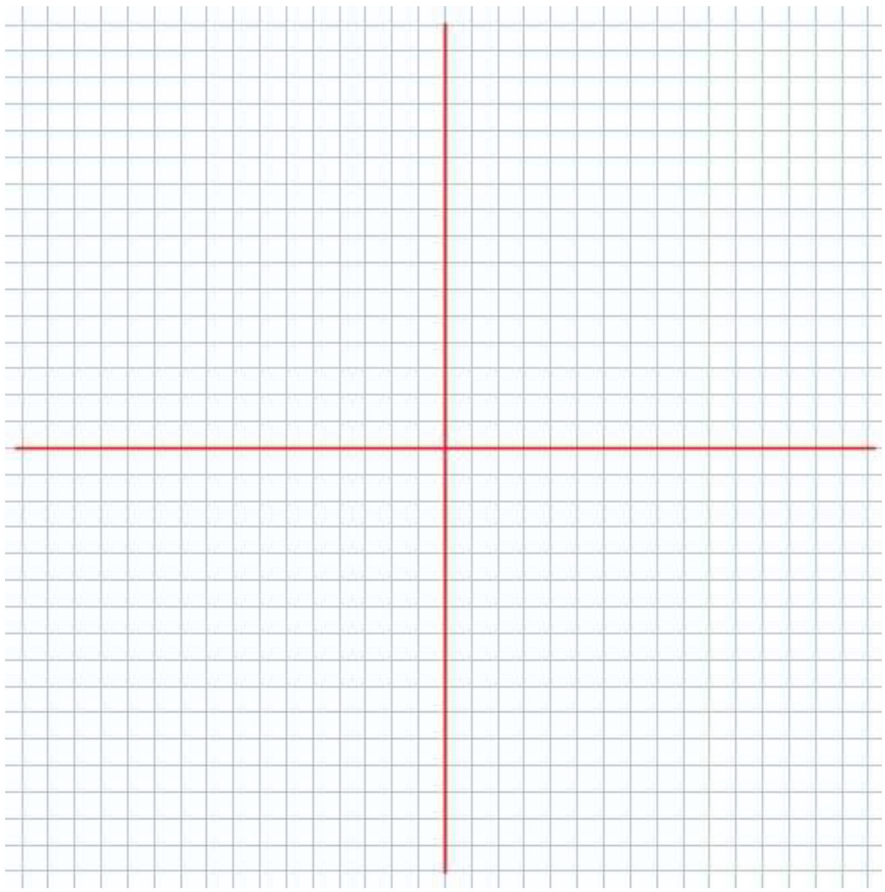
Given the sketch of the function $f(x)$ determine the image points of the transformed function $-2f\left(\frac{1}{3}(x+1)\right)+3$ and sketch the graph of the transformed function.



Example 1.8.6

On the same set of axes sketch the graphs of $f(x)=\sqrt{x}$ and $g(x)=2\sqrt{-x+1}-2$.

Determine three points on the parent function and state the image points for each.



Success Criteria:

- I can use the value of a to determine if there is a vertical stretch/reflection in the x-axis
- I can use the value of k to determine if there is a horizontal stretch/reflection in the y-axis
- I can use the value of d to determine if there is a horizontal translation
- I can use the value of c to determine if there is a vertical translation
- I can transform x coordinates by using the expression $\frac{1}{k}x + d$
- I can transform y coordinates by using the expression $ay + c$