Chapter 1 – Introduction to Functions

1.5: Inverses of Functions

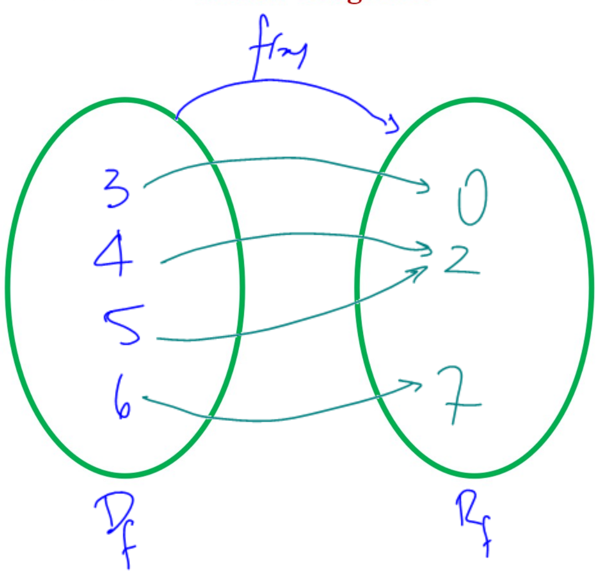
Learning Goal: We are learning to determine inverses of functions and investigate their properties.

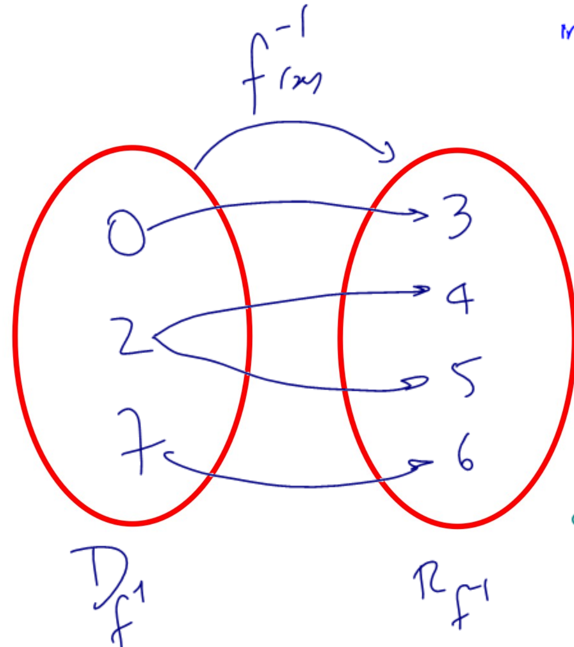
Definition 1.5.1 (*very rough definition!*)

Given a function $f(x)$, the inverse of the function (which we write as $f^{-1}(x)$) can be considered to "**undo**" what $f(x)$ originally did.

not an exponent
it simply indicates that it is an inverse relation

Consider the **Arrow Diagrams**:





Note $f^{-1}(x)$ is NOT a fn because the domain value $x=2$ is assigned to two range values (4 & 5) by $f^{-1}(x)$

Big Idea

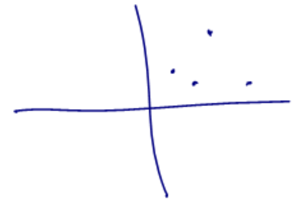
SWITCH DOMAIN & RANGE
(switch 'x' & 'y')

Note: $f(x)$ is an example of a <u>discrete</u> fn (a set of individual points)

Example 1.5.1

Given the graph of $f(x)$ determine: $D_f, R_f, f^{-1}(x), D_{f^{-1}}, R_{f^{-1}}$

$f(x) = \{(2,3),(4,2),(5,6),(6,2)\}$. Is $f^{-1}(x)$ a function?



$D_f = \{2,4,5,6\}$

switch x & y

$f^{-1}(x) = \{(3,2),(2,4),(6,5),(2,6)\}$

$R_f = \{3,2,6\}$

$D_{f^{-1}} = \{3,2,6\}$ $(= R_f)$

$R_{f^{-1}} = \{2,4,5,6\}$ $(= D_f)$

Note $f^{-1}(x)$ is not a fn (just a relation) because "$x=2$" is assigned to two range values

Determining the Inverse of a continuous Function

We can determine the inverse of some given function in either of two ways: Graphically and Algebraically.

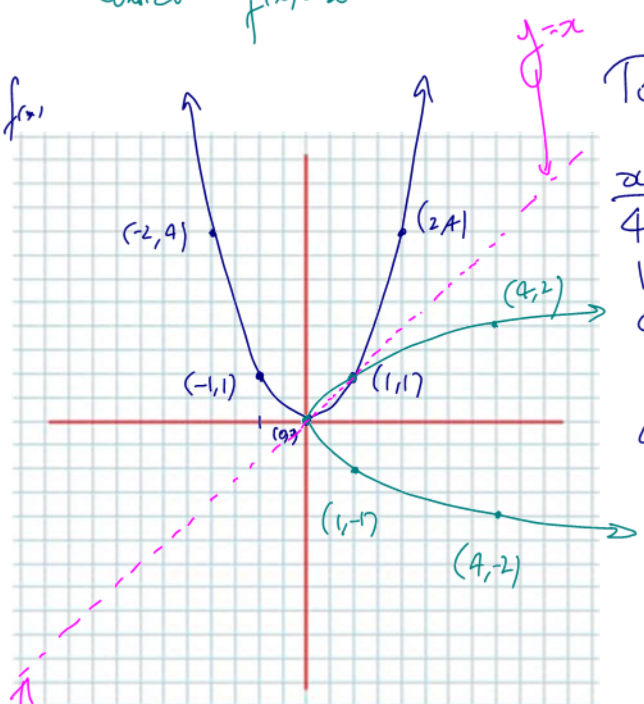
Note: Finding a function inverse graphically is not a very useful method, but it can be instructive.

Function Inverses Graphically

Consider $f(x) = x^2$

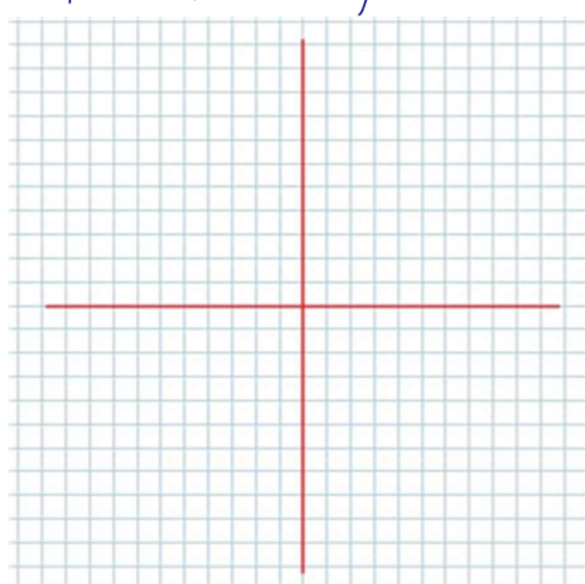
ToV for $f(x)$

x	$f(x)=x^2$
-2	4
-1	1
0	0
1	1
2	4



ToV for $f^{-1}(x)$ (switch x & y)

x	f^{-1}
4	-2
1	-1
0	0
1	1
4	2



invariant under inversion

acts like a mirror through which a fn and its inverse are reflections of each other

Function Inverses Algebraically

Determining algebraic representations of inverse relations for given functions can be done in (at least) two ways:

1) Use algebra in a "brute force" manner (keeping in mind the Big Idea)
2) Use Transformations (keeping in mind "inverse operations")

Example 1.5.2

Determine the inverse of

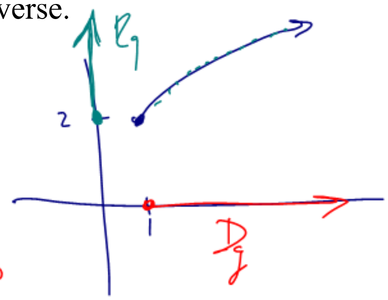
a) $f(x) = 2x - 5$ b) $g(x) = \frac{1}{2}\sqrt{x-1} + 2$.

(1, 2)

State the domain and range of both the function and its inverse.

> Here we will use "brute force".
>
> Method:
>
> 1) Switch x and $f(x)$, and call "$f(x)$", $f^{-1}(x)$.
> 2) Solve for $f^{-1}(x)$
>
> using SAMDEB

using inverse operations



a) $f(x) = 2x - 5$

inversion $\Rightarrow x = 2f^{-1}(x) - 5$ +5

$\Rightarrow x + 5 = 2f^{-1}(x)$ $\div 2$ on both $\boxed{\text{SIDES}}$

$\Rightarrow \frac{x+5}{2} = f^{-1}(x)$

$\Rightarrow f^{-1}(x) = \frac{x+5}{2}$

$= \frac{x}{2} + \frac{5}{2}$

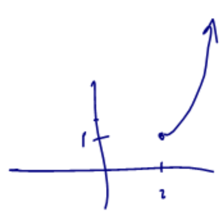
$f^{-1}(x) = \frac{1}{2}x + \frac{5}{2}$

b) $g(x) = \frac{1}{2}\sqrt{x-1} + 2$

$x = \frac{1}{2}\sqrt{g^{-1}(x) - 1} + 2$

$\Rightarrow x - 2 = \frac{1}{2}\sqrt{g^{-1}(x) - 1}$ $(\ )\times 2$ $\times 2$

$2(x-2) = \sqrt{g^{-1}(x) - 1}$ square both sides



$(2(x-2))^2 = g^{-1}(x) - 1$

$D_g = \{x \in \mathbb{R} \mid x \geq 1\}$ $D_{g^{-1}} = \{x \in \mathbb{R} \mid x \geq 2\}$ $\Rightarrow g^{-1}(x) = (2(x-2))^2 + 1$

$R_g = \{g(x) \in \mathbb{R} \mid g(x) \geq 2\}$ $R_{g^{-1}} = \{g^{-1}(x) \in \mathbb{R} \mid g^{-1}(x) \geq 1\}$

Method ① Determine the function type for the inverse

② Determine all transformations (and then invert them)

Example 1.5.3

Using transformations determine the inverse of $f(x) = 2\sqrt{\frac{1}{3}(x-1)} + 2$.

$$f^{-1}(x) = 3\left(\frac{1}{2}(x-2)\right)^2 + 1$$

Consider $g(x) = 3\left(-5(x+1)\right)^2 - 4$

$$g^{-1}(x) = \pm\frac{1}{5}\sqrt{\frac{1}{3}(x+4)} - 1$$

don't forget the "$\pm$"

Success Criteria:

- I can determine the inverse of a function using various techniques
- I can determine the inverse of a coordinate (a , b) by switching the variables: (b , a)
- I can recognize that the domain of an inverse is the range of the original function
- I can recognize that the range of an inverse is the domain of the original function
- I can understand that the inverse of a function is a reflection along the line y = x