$f(x) = 3x^2 - 5x + 2$

⇓

quadratic

Chapter 3 – Quadratic Functions

3.1 – Properties of Quadratic Functions

Learning Goal: We are learning to represent and interpret quadratic functions in three different forms.

This lesson is a review of some of what we learned about quadratics in Grade 10. In Grade 10 we studied the **THREE FORMS** of quadratic functions and the **information** they give:

1) Standard Form - $f(x) = ax^2 + bx + c$

$f(0) = a(0)^2 + b(0) + c = c$

Information

"c" = y intercept (recall the y int means $x = 0 \Rightarrow (0, f(0))$

"a" tells us which way the parabola opens.

Up means $a > 0$, Down means $a < 0$

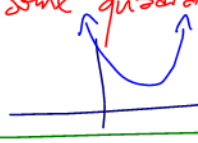
2) Zeros (or Factored) Form - $f(x) = a(x-r)(x-s)$

Information

"a" – tells us about the direction the parabola opens

$x = r$, $x = s$ are the zeros (x-intercept – when $f(x) = 0$) with coordinates $(r, 0)$ $(s, 0)$

Note: Some quadratics CANNOT be written in zeros form



this parabola does NOT cross the x-axis

⇒ No zeros ⇒ No zeros form.

3) Vertex Form - $f(x) = a(x-h)^2 + k$

Information

a – tells us the direction of opening.

vertex (h, k)

eg Consider $g(x) = -3(x+2)^2 - 7$

INFO

① the parabola opens down because $a = -3 < 0$

(proper form: $g(x) = -3(x-(-2))^2 - 7$, $h = -2$)

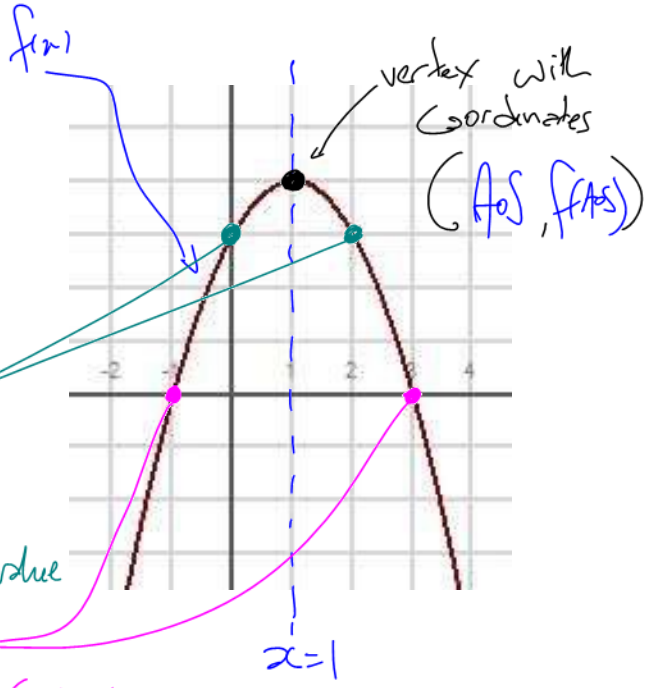
② The vertex is $(-2, -7)$

AoS

Recall the concept of the axis of symmetry.

The AoS is a vertical line $x = \#$ passing through the vertex

The AoS equation $x = \#$



These pairs of points have the same y-value

This pair of points (zeros) have the same y-value

$\Rightarrow$ we call such pairs of points Symmetric partners. (they share a y-value)

Example 3.1.1

Given the quadratic function $f(x) = \frac{1}{2}(x+3)^2 - 1$, state:

$f(x) = a(x-h)^2 + k$ → vertex

a) The direction the parabola opens

b) The coordinates of the vertex

c) The equation of the axis of symmetry

a) opens up because $a = \frac{1}{2} > 0$

b) vertex $(-3, -1)$ $(AoS, f(AoS))$

c) $x = -3$

$g(-1) = -2(-1+3)(-1-1)$
$= -2(2)(-2) = +8$

zeros.

$g(0) = -2(0+3)(0-1)$
$= +6$

Example 3.1.2

Given the quadratic function $g(x) = -2(x+3)(x-1)$, state

a) The direction the parabola opens
b) The zeros of the quadratic
c) The equation of the axis of symmetry
d) The coordinates of the vertex
e) The function in vertex form

Sketch the graph of the function.

d) vertex is $(AoS, g(AoS))$
$= (-1, g(-1))$
$= (-1, 8) = (h, k)$

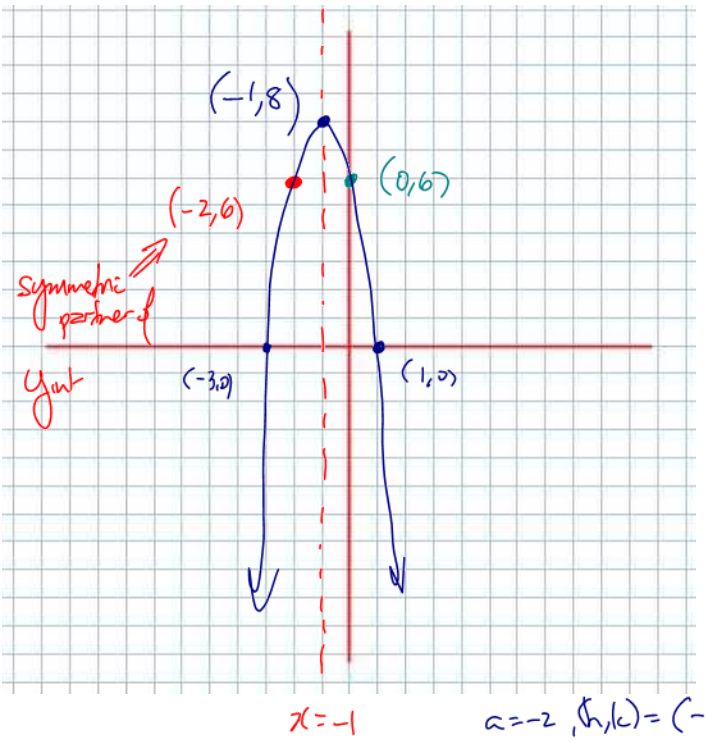
a) down because $a = -2 < 0$

b) zeros are $x = -3$, $x = +1$

c) The zeros are symmetric partners
$\Rightarrow$ the AoS is the average of the x-values of the zeros

$\therefore$ AoS: $x = \frac{-3+1}{2} = \frac{-2}{2} = -1$

$\therefore$ $x = -1$



just need $x = 0$
$(0, g(0))$
$(0, 6)$

$x = -1$ $\quad a = -2, (h,k) = (-1, 8)$

e) Vertex form: $g(x) = a(x-h)^2 + k$. $\quad g(x) = -2(x+1)^2 + 8$.

Example 3.1.3

Given the two points $(4,7), (-5,7)$ which are on a parabola, determine the equation of the axis of symmetry.

same y-value $\Rightarrow$ symmetric partner points!

$\Rightarrow$ the AoS is the average of the domain values of the partners

eg AoS: $x = \frac{4+(-5)}{2} = -\frac{1}{2}$ $\quad x = -\frac{1}{2}$

$h(4) = -4(4)^2 + 32(4) = -64 + 128 = 64$ $\quad h(t) = -4t^2 + 32t$

Example 3.1.4 (From Pg. 147 in your text)

11. The height of a rocket above the ground is modelled by the quadratic function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

a) Graph the quadratic function. NEED INFO

b) How long will the rocket be in the air? How do you know?

c) How high will the rocket be after 3 s?

d) What is the maximum height that the rocket will reach?

y-int: $(0, h(0)) = (0, 0)$

$h(0) = -4(0)^2 + 32(0)$

$= 0$

Zeros: (converting to factored form $\Rightarrow$ factor)

$h(t) = -4t(t-8)$ $\quad$ ($= -4(t-0)(t-8)$)

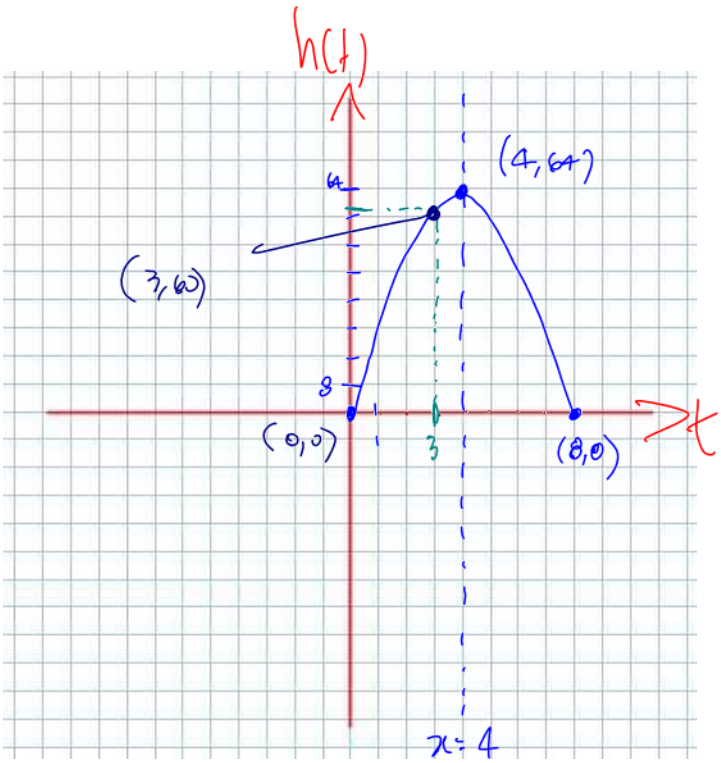
zeros are $t = 0$, $t = 8$

vertex $= (AoS, h(AoS))$

AoS: $x = \frac{0+8}{2} \Rightarrow x = 4$

$\Rightarrow$ vertex is

$(4, h(4)) = (4, 64)$



b) 8 seconds (the domain is $0 \le x \le 8$)

c) $h(3) = -4(3)^2 + 32(3)$

$= 60$ m

d) 64 m. (the y value of the vertex)

Success Criteria:

- I can recognize a quadratic function in standard, factored, and vertex form
- I can determine the zeros, direction of opening, axis of symmetry, vertex, domain and range from the graph of a parabola
- I can determine the equation of quadratic function from its parabola