VERTEX

Chapter 3 – Quadratic Functions

3.2 – The Maximum or Minimum of Quadratic Functions

Learning Goal: We are learning to determine the maximum/minimum value of a quadratic function.

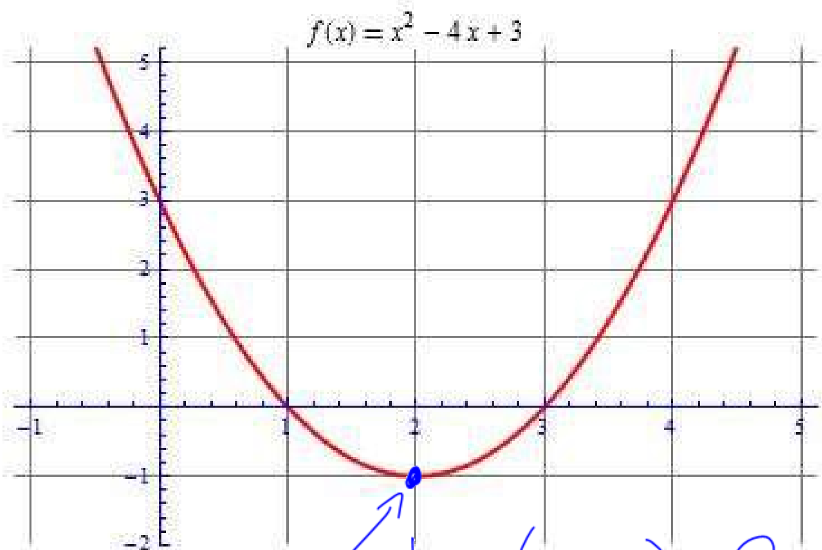
One very important aspect of quadratic functions is that they all have either a maximum (if the associated parabola opens down) or a minimum (if the parabola opens up). **Max/Min's** have so many **applications** in the real world that it's **ridiculous**.

The **BIG QUESTION** we are faced with is this:

How do we find the Maximum or Minimum Value for some given Quadratic?

Example 3.2.1

To find a minimum (or maximum) of a quadratic, **you are NOT allowed to**



f(x) has a min of −1 at x = 2

vertex (2, −1) This CONTAINS THE MIN

Figure 3.2.1

Figure 3.2.1b

So, we do need to find the vertex, but we also need to **KEEP IN MIND WHAT THE NUMBERS ASSOCIATED WITH THE VERTEX MEAN**.

(AoS, f(AoS))

In order to find the vertex using algebra, we will consider three techniques:

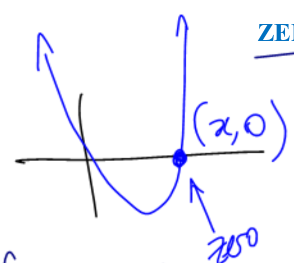
1) **USING THE ZEROS, TO FIND THE AXIS OF SYMMETRY**, and then the vertex (**this is the easiest technique**, ***assuming we can factor the quadratic***).
2) **COMPLETING THE SQUARE** to find the vertex (this is the toughest technique, but it's nice because you ***end up with the quadratic in vertex form***).
3) ☆ **USE PARTIAL FACTORING TO FIND THE AXIS OF SYMMETRY**, and then the vertex.
4) $AoS = -\frac{b}{2a}$ from $ax^2+bx+c = f(x)$

Note: We can also use graphing calculators to find the max/min of a quadratic!

Example 3.2.2

we are looking for a max → factoring.

Determine the max or min value for the function $f(x) = -3x^2 - 12x + 15$ by finding **THE ZEROS** of the quadratic.



$f(-2) = -3(-2)^2 - 12(-2) + 15 = 27$

finding zeros → $f(x) = 0$

$\Rightarrow -3x^2 - 12x + 15 = 0 \quad \div -3$

$x^2 + 4x - 5 = 0$

x	+
-5	4
5, -1	

$(x+5)(x-1) = 0$

$\Rightarrow x = -5, +1$

$\therefore$ The AoS: $x = \frac{-5+1}{2} = -2$

$\therefore$ The vertex is $(-2, f(-2))$

$= (-2, 27)$

$\therefore$ The max is 27 at $x = -2$.

Example 3.2.3

COMPLETE THE SQUARE to find the vertex of the quadratic and state **where** the max (min) is and **what** the max (min) is.

$$g(x) = 2x^2 + 8x - 5$$

we will look for a min since $a = +2$.

$g(x) = 2(x^2 + 4x) - 5$

$\left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$\Rightarrow g(x) = 2(\underbrace{x^2 + 4x + 4}_{\text{perfect square}} - 4) - 5$

$\Rightarrow g(x) = 2((x+2)^2 - 4) - 5$

$\Rightarrow g(x) = 2(x+2)^2 - 8 - 5$

$\Rightarrow g(x) = 2(x+2)^2 - 13$

vertex form!!!

$\therefore$ The vertex is $(-2, -13)$

$\boxed{\therefore g(x) \text{ has a min of } -13 \text{ at } x = -2}$

Algorithm

① Factor "a" from the 1st two terms (leave the constant alone)

② Take the "new "b"" and a) divide by 2 then b) square it

③ Add that new squared # on and then subtract it off

④ The 1st 3 terms (inside the bracket) are now a perfect square symmetric partners

$h(0) = -2 \Rightarrow (0, -2)$ || $h(-3) = -2 \Rightarrow (-3, -2)$

Example 3.2.4

Using **PARTIAL FACTORING** determine the axis of symmetry. Then find the vertex and state the min or max value.

$$h(x) = 5x^2 + 15x - 2$$

Since $a = +5$ we will find a min

step ①

$\Rightarrow h(x) = 5x(x+3) - 2$

AoS: $x = \dfrac{0 + (-3)}{2} = -1.5$

calculate

$\therefore$ Vertex is $(-1.5, h(-1.5))$

$= (-1.5, -13.25)$

$\therefore$ $h(x)$ has a min of -13.25 at $x = -1.5$.

step ②

$5x(x+3) = 0$

$x = 0, -3$

$h(1.5)$

$= 5(-1.5)^2 + 15(-1.5) - 2$

$= -13.25$

Algorithm

① Fully factor part of the fn (the 1st two terms – leave the constant alone.

② Set the partial factored bit to zero & get x's

③ Use the x's from ② to find the AoS

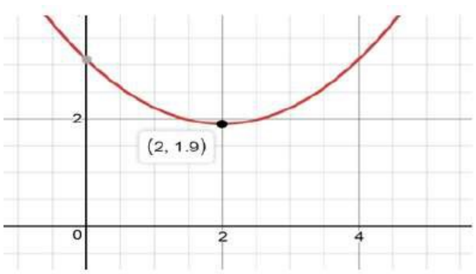
④ (AoS, f(AoS))

Example 3.2.5

Using graphing technology, determine the max/min value of the quadratic

$a=0.3$ $b=-1.2$ $c=3.1$

$$f(x) = 0.3x^2 - 1.2x + 3.1$$



$f(2) = 0.3(2)^2 - 1.2(2) + 3.1 = 1.9$

Using AoS $= -\dfrac{b}{2a}$

$\therefore$ AoS $= -\dfrac{-1.2}{2(0.3)} = \dfrac{1.2}{0.6} = 2$

$\therefore$ The vertex is

$(2, f(2)) = (2, 1.9)$

$\therefore$ The min is 1.9 at

$x = 2$

Success Criteria:

- I can recognize when a function has a maximum or minimum value (based on "*a*")
- I can find the max/min (vertex) value using various methods (*partial factoring* ☺)