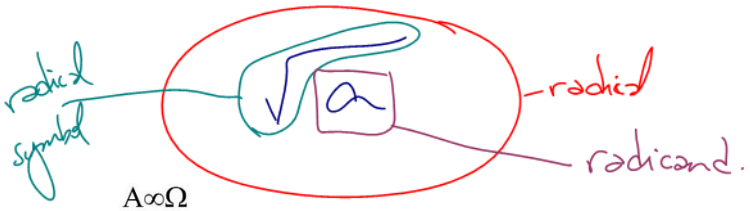

Chapter 3 – Quadratic Functions

3.4 – Operations with Radical Numbers

Learning Goal: We are learning to simplify and perform operations on radicals.

First we need to understand that **RADICALS** (*square roots, cube roots, etc*) **ARE NUMBERS**, and working with them should not induce any kind of fear in your spirit. So, **FEAR NOT!**

A COUPLE OF THINGS TO REMEMBER:

1) The square root of a square number is a nice integer.

e.g. $\sqrt{25} =$ 5 because $5^2 = 25 \Rightarrow \sqrt{5^2} = \sqrt{25}$ the square and the square root "cancel"

$\sqrt{49} =$ 7

2) The cube root of a cubed number is a nice integer

e.g. $\sqrt[3]{27} =$ 3 because $3 \times 3 \times 3 = 27 \Rightarrow 3^3 = 27$

$\Rightarrow 3 = (27)^{\frac{1}{3}} = \sqrt[3]{27}$

$\sqrt[3]{125} =$ 5 because $5 \times 5 \times 5 = 125 \Rightarrow 5^3 = 125$.

Now, if we don't have a radical with a perfect square (or cube as the case may be) we could use a calculator to find the root.

e.g. $\sqrt{24} = 4.898979485566356196394568114941 18...$

BUT the "**DECIMAL EXPANSION**" is **unending** and **doesn't repeat** and so we can only **APPROXIMATE THE VALUE** of $\sqrt{24}$ because of the need to **ROUND-OFF**. "**EXACT NUMBERS**" like $\sqrt{24}$ are sometimes preferred in mathematical solutions and so **we do need to know how to work with these radical NUMBERS.** Working with radical numbers means we'll be:

- adding/subtracting
- multiplying/dividing them.

Before beginning, there is one thing to keep in mind:

2 root 5

1	4	9	16	25	36	49	64	81	100	
1	2	3	4	5	6	7	8	9	10	...

COEFFICIENTS WITH COEFFICIENTS, RADICALS WITH RADICALS

e.g. The number $2\sqrt{5}$ has a coefficient part of 2 and a radical part of $\sqrt{5}$

Such a number (with both a coefficient and a radical part) is called a mixed radicals

Example 3.4.1

Multiply the following:

a) $\sqrt{5}\times\sqrt{3}$ $= \sqrt{5\times3} = \sqrt{15}$

b) $-2\sqrt{7}\times3\sqrt{6}$ $= (-2\times3)\sqrt{7\times6} = -6\sqrt{42}$

c) $5\sqrt{10}\times\sqrt{5}$ $= 5\sqrt{50}$ ($\sqrt{10}\times\sqrt{5}$; 5×1)

d) $\sqrt{2}\times\sqrt{2}$ $= \sqrt{4} = 2$

$(\sqrt{2})^2 = 2$.

Rules

$\sqrt{a}\times\sqrt{b} = \sqrt{ab}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$

Example 3.4.2

Simplify the following:

search for perfect square factors

a) $\sqrt{50} = \sqrt{25\times2} = \sqrt{25}\times\sqrt{2} = 5\sqrt{2}$

b) $-2\sqrt{27} = -2\sqrt{9\times3} = -2(\sqrt{9})(\sqrt{3}) = -2(3)\sqrt{3} = -6\sqrt{3}$.

c) $2\sqrt{50}\times\left(-3\sqrt{24}\right)$

2 methods

① Go for it and then simplify

$= (2)(-3)(\sqrt{50})(\sqrt{24})$

$= -6\sqrt{1200}$

$= -6\sqrt{100\times12} = -6(10)\sqrt{12} = -60\sqrt{12}$

$= -60\sqrt{4\times3}$

$= -60(2)\sqrt{3}$

$= -120\sqrt{3}$

② Simplify 1st, then multiply

$= (2\sqrt{25\times2})\times(-3\sqrt{4\times6})$

$= 2(5)\sqrt{2}\times(-3)(2)\sqrt{6}$

$= 10\sqrt{2}\times-6\sqrt{6}$

$= -60\sqrt{12} = -60\sqrt{4\times3}$

$= -60(2)\sqrt{3} = -120\sqrt{3}$

by adding/subtracting COEFFICIENTS

Note: **We can only ADD OR SUBTRACT "LIKE" RADICALS.**
e.g. $2\sqrt{3}$ and $-5\sqrt{3}$ **ARE LIKE, but** $2\sqrt{5}$ and $3\sqrt{20}$ **ARE NOT** ***(or aren't they?.....)***

Example 3.4.3

Add the following:

a) $3\sqrt{2} + 7\sqrt{2}$

$= 10\sqrt{2}$

add

b) $5\sqrt{7} - 3\sqrt{5} - 7\sqrt{7}$

not like the others

$= 5\sqrt{7} - 7\sqrt{7} - 3\sqrt{5}$

$= -2\sqrt{7} - 3\sqrt{5}$ (no more simplifying)

c) $2\sqrt{5} - 3\sqrt{20}$ (simplify!)

$= 2\sqrt{5} - 3\sqrt{4\times5}$

$= 2\sqrt{5} - 3(2)\sqrt{5}$

$= 2\sqrt{5} - 6\sqrt{5} = -4\sqrt{5}$

d) $-3\sqrt{300} + \sqrt{243}$

$= -3\sqrt{100\times3} + \sqrt{81\times3}$

$-3\times\sqrt{100}$
$= -3\times10$

$= -30\sqrt{3} + 9\sqrt{3}$

$= -21\sqrt{3}$

Example 3.4.4 (use the distributive property)

Simplify:

a) $2\sqrt{3}\ (3\sqrt{2} - 5\sqrt{6})$

$= (2\sqrt{3})(3\sqrt{2}) - (2\sqrt{3})(5\sqrt{6})$

$= 6\sqrt{6} - 10\sqrt{18}$ "18" has a perfect square factor "9"

$= 6\sqrt{6} - 10\sqrt{9\times 2} = 6\sqrt{6} - 30\sqrt{2}$

$10\times\sqrt{9} = 10\times 3 = 30.$

b) $(3\sqrt{12} - 5\sqrt{2})(2\sqrt{3} + 3\sqrt{2})$ FOIL.

$= 6\sqrt{36} + 9\sqrt{24} - 10\sqrt{6} - 15\sqrt{4}$

$= 6(6) + 9\sqrt{4\times 6} - 10\sqrt{6} - 15(2)$ $9\times\sqrt{4} = 9\times 2$

$= 36 + 18\sqrt{6} - 10\sqrt{6} - 30$ $18\sqrt{6} - 10\sqrt{6}$

$= 6 + 8\sqrt{6}$ $36 - 30$

c) $(5 - 2\sqrt{2})^2$

$= (5 - 2\sqrt{2})(5 - 2\sqrt{2})$

$= 25 - 10\sqrt{2} - 10\sqrt{2} + 4\sqrt{4}$

$= 25 - 20\sqrt{2} + 4(2)$

$25 - 20\sqrt{2} + 8$

$= 33 - 20\sqrt{2}$

Success Criteria:

- I can recognize "like" radicals.
- I can write a radical in simplest form
- I can simplify radicals by adding, subtracting, multiplying, and dividing
- I can appreciate that a radical is an EXACT answer.