

Chapter 3 – Quadratic Functions

3.6 – Zeroes of Quadratic Functions

Learning Goal: We are learning to determine the number of zeros of a quadratic function.

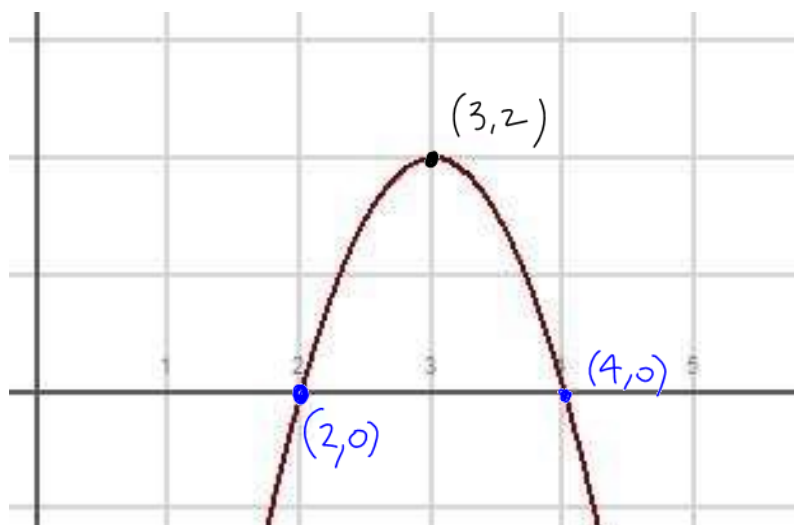
Before we begin, let's think about a couple of things...

Remember – **FUNCTIONS CAN BE DESCRIBED AS A SET OF ORDERED PAIRS**, where the “ordered pair” is a pair of numbers: a **domain value** and a **range value** which can look like $(x, f(x))$. We have talked about the vertex of a parabola. Consider a parabola opening down (which means it will have a maximum value).

The vertex of that parabola is NOT the maximum. Instead, the vertex is a POINT which is made up of two special numbers. The domain value is WHERE the max occurs and the functional value (the “y” value) is the maximum.

When we talk about the ZEROS of a quadratic we need to understand what we mean by that.

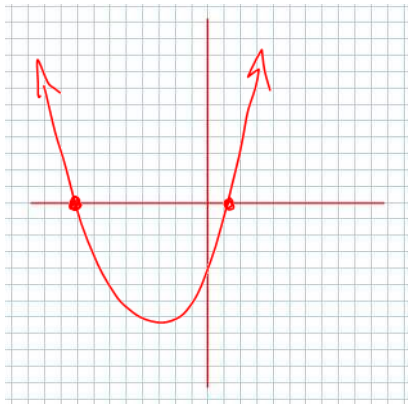
Consider the sketch of the graph of the quadratic function $f(x) = -2(x-3)^2 + 2$



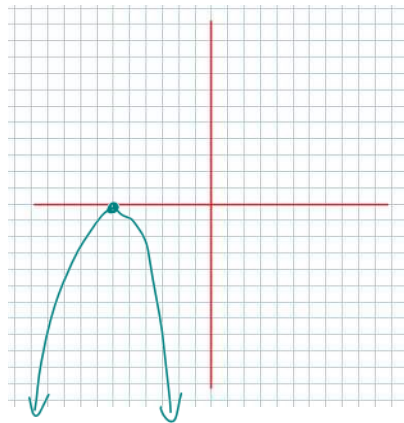
Zero is/are the location(s)
where the function value
(y value) is zero

$f(x)$ has two zeros at
 $x = 2, 4$

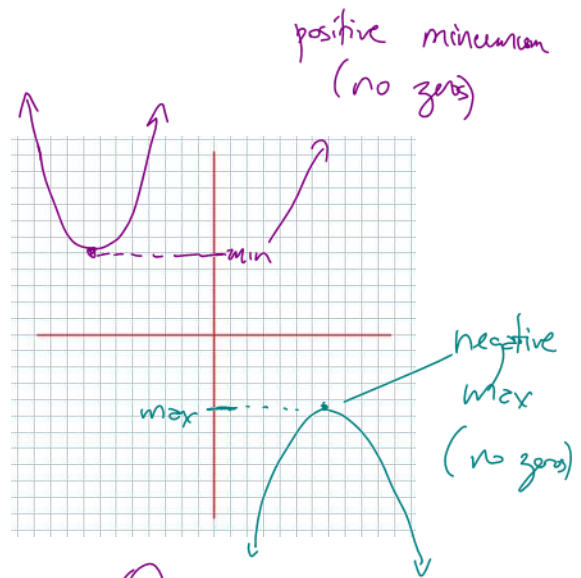
Q. Do all quadratics have 2 zeros? NO!!!!!!!



2 zeros



1 zero (vertex is on the domain axis)



0 zeros

Finding the Zeros of a Quadratic

We can find those pesky zeros in a number of ways:

- 1) Writing the quadratic in zeros form (by factoring)
- 2) Writing the quadratic in vertex form, and doing some algebra (a bit nasty)
- 3) Using the quadratic formula (but the quadratic **MUST BE IN STANDARD FORM** -

$$f(x) = ax^2 + bx + c$$

- 4) Using graphing technology (lame, but legit)

Example 3.6.1

Determine the zeros:

a) $f(x) = x^2 - 3x - 4$

zeros occur when $f(x) = 0$

Consider $x^2 - 3x - 4 = 0$

x	+
-4	-3
	-4

$$\Rightarrow (x+1)(x-4) = 0$$

$\therefore x = -1, 4$ are the zeros

b) $g(x) = 2x^2 + x - 1$

$$2x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

Consider $2x^2 + x - 1 = 0$

x	+
-2	+1
	-1, 2

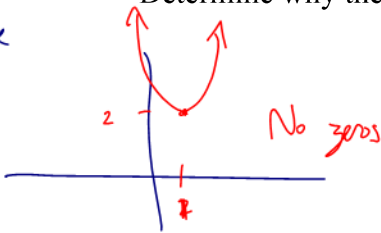
$$(2x-1)(x+1) = 0$$

\therefore the zeros are $x = \frac{1}{2}, -1$

Example 3.6.2

Determine why the quadratic $f(x) = 2(x-1)^2 + 2$ has no zeros.

picture



The vertex is $(1, 2)$
 $f(1) = 2$ is $>$ min. because

$f(x)$ opens up
 Since there is a positive min
 \Rightarrow No zeros

Example 3.6.3

Determine the zeros of $g(x) = -(x+1)^2 + 8$

Consider $-(x+1)^2 + 8 = 0$

$\Rightarrow \overset{\div -1}{-(x+1)^2} = -8$

$\Rightarrow (x+1)^2 = 8$

square root both sides

$x+1 = \pm \sqrt{8}$

$\Rightarrow x = \pm \sqrt{8} - 1$

There are TWO ZEROS
 WRITE THEM BOTH DOWN

$\Rightarrow x = \sqrt{8} - 1 \quad \text{or} \quad x = -\sqrt{8} - 1$

$\sqrt{8} = \sqrt{4 \times 2}$
 $= (\sqrt{4})(\sqrt{2})$
 $= 2\sqrt{2}$

$\Rightarrow x = 2\sqrt{2} - 1 \quad \text{or} \quad x = -2\sqrt{2} - 1$

- with radicals is called EXACT FORM

$\Rightarrow x \approx 1.83 \quad \text{or} \quad x \approx -3.83$

- approximate form (rounded form)

or
 rounding to 2 decimal places

Example 3.6.4

Using the quadratic formula, determine the zeros of the quadratic:

In case you've forgotten, the quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

a) $f(x) = 2x^2 + 3x - 7$

b) $g(x) = 3x^2 - 2x + 4$

Consider $2x^2 + 3x - 7 = 0$
 $a=2$ $b=3$ $c=-7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-7)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{65}}{4}$$

\therefore The two zeros are

$$x = \frac{-3 + \sqrt{65}}{4} \quad \text{or} \quad x = \frac{-3 - \sqrt{65}}{4}$$

(exact)

$$\doteq 1.27 \quad \text{or} \quad \doteq -2.77$$

(approximate)

Consider $3x^2 - 2x + 4 = 0$
 $a=3$ $b=-2$ $c=4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 - 48}}{6}$$

$$= \frac{2 \pm \sqrt{-44}}{6}$$

\therefore No zeros

$(\text{we cannot square root a negative \#})$

Recall - Quadratic fns can have 0, 1 or 2 zeros



$$f(x) = ax^2 + bx + c$$

The Discriminant

QF.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Discriminant of the quadratic formula is called the **DISCRIMINANT** because it discriminates between the 3 scenarios of quadratic zeros (0, 1, or 2 zeros)

The Discriminant is $b^2 - 4ac$

- 1) If $b^2 - 4ac > 0$ \Rightarrow the quadratic has 2 zeros $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
(positive)
- 2) If $b^2 - 4ac = 0 \Rightarrow$ 1 zero: $x = \frac{-b}{2a}$ (recall $x = \frac{-b}{2a}$ is the AOS!)
- 3) If $b^2 - 4ac < 0 \Rightarrow$ 0 zeros
(negative)

Example 3.6.5

Determine the number of zeros using the discriminant:

a) $f(x) = 2x^2 + 3x - 2$
 $a=2 \quad b=3 \quad c=-2$

$$\begin{aligned} b^2 - 4ac &= (3)^2 - 4(2)(-2) \\ &= 9 - (-16) \\ &= +25 > 0 \Rightarrow f(x) \text{ has 2 zeros} \end{aligned}$$

b) $g(x) = -x^2 + 4x - 4$
 $a=-1 \quad b=4 \quad c=-4$

$$\begin{aligned} b^2 - 4ac &= (4)^2 - 4(-1)(-4) \\ &= 16 - 16 \\ &= 0 \Rightarrow 1 \text{ zero for } g(x) \end{aligned}$$

c) $h(x) = 3x^2 + 5x + 6$
 $a=3 \quad b=5 \quad c=6$

$$\begin{aligned} b^2 - 4ac &= (5)^2 - 4(3)(6) \\ &= 25 - 72 \\ &= -47 < 0 \therefore 0 \text{ zeros} \end{aligned}$$

Success Criteria:

- I can recognize that a quadratic function may have 0, 1, or 2 zeros
- I can use the discriminant of the quadratic formula to determine the number of zeros

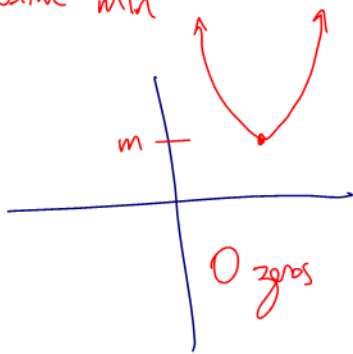
Find # of zeros

① If in standard form ($f(x) = ax^2 + bx + c$)
→ use the discriminant

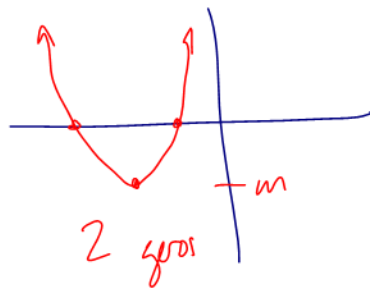
② If in zeros form - just look at it! | eg $g(x) = -2(x+1)(x-5)$
→ 2 zeros

③ If in vertex form - use the following 4 scenarios

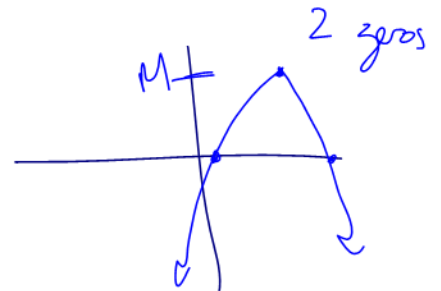
positive min



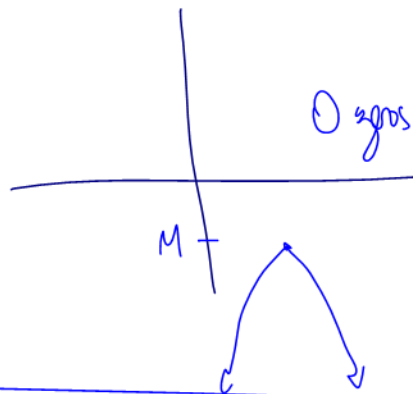
negative min



positive Max



negative Max



Note forms can be tricky

eg $f(x) = 2x^2 - 5$

standard but also vertex

$f(x) = 2(x-0)^2 - 5$ → vertex is $(0, -5)$

eg $g(x) = -3(x-5)^2$

zeros form but also vertex

$g(x) = -3(x-5)^2 + 0$ vertex $(+5, 0)$