Chapter 3 – Quadratic Functions

3.5 – Solving Quadratic Equations

Learning Goal: We are learning to solve quadratic functions in different ways.

Note that last day we looked at section 3.6. We now go back to 3.5 as this is a better order for the concepts.

→ made of points $(x, f(x))$ and there are infinitely many of them

Before beginning we should look at the difference between a Quadratic **FUNCTION** and a Quadratic **EQUATION**. A function such as $f(x) = 3x^2 - 5x + 1$ has a graph with **infinitely** many points. On the other hand, a quadratic equation (in standard form) looks like:

$$\boxed{3x^2 - 5x + 1 = 0}$$

equations have a finite number of solutions

(*What is the difference between the function and the equation?*)

In section 3.6 we saw how to find the **zeroes** of quadratic functions, using the techniques of factoring, the quadratic formula or using graphing technology. As it turns out, solving a quadratic equation is **Exactly** the same as finding zeros of quadratic fns

Quadratic equations, therefore can have 0, 1, or 2 **SOLUTIONS**/roots/~~zeros~~

all mean the same thing

zeros usually reserved for fns

Example 3.5.1

Solve the equations:

a) $1x^2 - 5x - 14 = 0$

x	+
−14	−5

2, −7

$(x-7)(x+2) = 0$

$x-7=0$ or $x+2=0$

$\Rightarrow x = 7, -2$

b) $2x^2 + 5x = 2x + 4$

quadratic

$-2x - 4$ $\quad -2x - 4$

we must be in "standard form"

"stuff = 0"

$\rightarrow ax^2 + bx + c = 0$

x	+
−8	+3

X d.n.f. ⇒ QF

$\Rightarrow 2x^2 + 3x - 4 = 0$

$a = 2 \quad b = 3 \quad c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{41}}{4}$$

$\therefore x = \frac{-3 + \sqrt{41}}{4} \doteq 0.85$

or

$= \frac{-3 - \sqrt{41}}{4} \doteq -2.35$

(exact answers)

approx. answers

Example 3.5.2

because of decimals – Q.F.

No "exact" answers found right away

Solve $-2.3x^2 - 1.32x = -1.45$ $+1.45$ $+1.45$

$$\Rightarrow -2.3x^2 - 1.32x + 1.45 = 0$$

$a=-2.3$ $b=-1.32$ $c=1.45$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{1.32 \pm \sqrt{(-1.32)^2 - 4(-2.3)(1.45)}}{2(-2.3)}$$

$$= \frac{1.32 \pm 3.88}{-4.6}$$

$$\therefore x = \frac{1.32+3.88}{-4.6} \quad \text{or} \quad \frac{1.32-3.88}{-4.6}$$

$$\doteq -1.13 \qquad \doteq 0.56$$

Example 3.5.3 (From your text: Pg. 178 #6a)

"real world problem" ⇒ we must interpret the "answers" to make sense in the problem.

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.

a) $P(x) = -x^2 + 12x + 28$

mean together

$P(x) = 0$

We wish to solve for $P(x) = 0$

$$\Rightarrow -x^2 + 12x + 28 = 0 \quad x \text{ "}-1\text{"}$$

to make factoring easier we want "a" to be positive

x "-1"

$$\Rightarrow x^2 - 12x - 28 = 0$$

×	+
−28	−12

2, −14

$$(x-14)(x+2) = 0$$

$$x = 14 \qquad x = -2$$

inadmissable (selling −2000 things makes no sense)

$\therefore$ If we sell 14 000 things we break even.

8. The population of a region can be modelled by the function $P(t) = 0.4t^2 + 10t + 50$, where $P(t)$ is the population in thousands and t is the time in years since the year 1995. $\Rightarrow$ in 1995, $t=0$

a) What was the population in 1995?

b) What will be the population in 2010?

c) In what year will the population be at least 450 000? Explain your answer.

$t=0$!

a) $P(0) = 0.4(0)^2 + 10(0) + 50$

$= 50$

$\therefore$ There are 50 000 people in 1995

b) 2010 $\Rightarrow$ $t = 2010 - 1995 = 15$

$P(15) = 0.4(15)^2 + 10(15) + 50$

$= 290$

$\Rightarrow$ in 2010 the population is 290 000 people

c) we want "t" when

$P(t) = 450$

$\Rightarrow 450 = 0.4t^2 + 10t + 50$ (−450, −450)

$\Rightarrow 0.4t^2 + 10t - 400 = 0$

$a = 0.4 \quad b = 10 \quad c = -400$

$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-10 \pm \sqrt{(10)^2 - 4(0.4)(-400)}}{2(0.4)}$

$= \frac{-10 \pm \sqrt{740}}{0.8}$

$\therefore t = \frac{-10 + \sqrt{740}}{0.8}$ or $\frac{-10 - \sqrt{740}}{0.8}$ inadmissible (no negative time)

$= 21.5$

1995 + 21.

$\therefore$ After 21.5 years (in 2016) the population is about 450 000

Success Criteria:

- I can solve quadratic functions by factoring, then setting each factor equal to zero
- I can solve quadratic functions by using the quadratic formula