Chapter 3 – Quadratic Functions

3.8 – Linear-Quadratic Systems

Learning Goal: We are learning to solve problems involving the intersection of a linear and quadratic function.

Recall from Grade 10 that solving a **S**YSTEM **OF** **L**INEAR **E**QUATIONS could be interpreted to mean finding the point of intersection of the two lines. The solution to a **SoLE** is a point, (x, y). From an algebraic point of view, we have two techniques for solving a SoLE:

1) Substitution
2) Elimination

Example 3.8.1

Solve the SoLE

$$2x + 3y = 7 \quad (1)$$
$$x - 2y = -7 \quad (2)$$

($+2y$ $+2y$)

Algebraically

Graphically

solution $(-1, 3)$

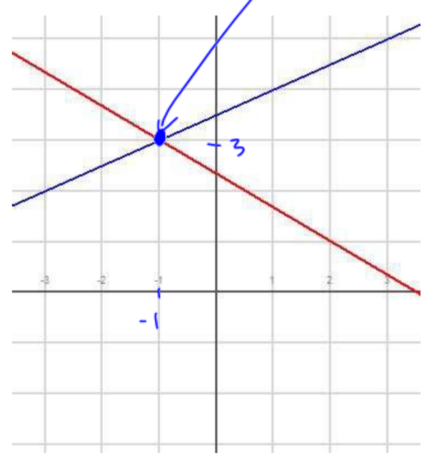

Figure 3.8.1

Substitution.

② $\Rightarrow$ $x = 2y - 7$ ③

Sub ③ into ①

$2(2y-7) + 3y = 7$

$4y - 14 + 3y = 7$ ($+14$ $+14$)

$7y = 21$

$y = 3$ sub into ③

$x = 2(3) - 7$

$= 6 - 7$

$= -1$

$\therefore$ The soln is $(x, y) = (-1, 3)$

Elimination

$2x + 3y = 7$ ①

$x - 2y = -7$ ②

② $\times 2 \rightarrow$ $2x - 4y = -14$ ③

① − ③ $\Rightarrow$ $7y = 21$

$3y - (-4y)$ $\quad 7 - (-14)$

$y = 3$ sub into ②

$x - 2(3) = -7$

$x - 6 = -7$

$x = -1$

$\therefore$ our soln is $(-1, 3)$

Solving a Linear-Quadratic System is more difficult, but we have the tools to succeed!
We will need to make use of (at least) one Property (or Rule) of Algebra:

THE TRANSITIVE PROPERTY OF EQUALITY

Rule: Given three numbers (or more generally, three mathematical objects) a, b, and c, and **if** $c = a$ and $c = b$, **then** $a = b$.

Example: If $f(x) = -2x - 4$, and $g(x) = x^2 - 3x - 10$, and if $f(x) = g(x)$, then

$$x^2 - 3x - 10 = -2x - 4$$

Note: **Solving a Linear-Quadratic System** is equivalent to **finding the solution(s) to a quadratic equation**. For L-QS`s we can therefore have 0, 1, or 2 solutions.

We will apply the techniques for solving quadratic equations!

Note: solutions are $\boxed{\text{points}}$ (x, y)

(use the line for the y-value)

Example 3.8.2

Solve the Linear-Quadratic System given directly above.

"stuff = 0"

$$x^2 \overset{+2x}{-3x} \overset{+4}{-10} = \overset{+2x}{-2x} \overset{+4}{-4}$$

$$\Rightarrow x^2 - x - 6 = 0$$

$\times$	$+$
-6	-1

$-3, +2$

$$(x-3)(x+2) = 0$$

$$\therefore x = 3 \text{ or } x = -2$$

$$f(3) = -2(3) - 4 = -6 - 4 = -10$$

$$f(-2) = -2(-2) - 4 = 4 - 4 = 0$$

$\therefore$ The solns are $(3, -10)$ or $(-2, 0)$

Example 3.8.3 (#2c, on Page 198 from your text)

Determine the point(s) of intersection of the two functions algebraically:

$f(x) = 3x^2 - 2x - 1, \ \ g(x) = -x - 6$

System: $3x^2 - 2x - 1 = -x - 6$

$\times$	$+$
15	-1

$\Rightarrow 3x^2 - x + 5 = 0$

$a = 3 \quad b = -1 \quad c = 5$

d.n.f. $\Rightarrow$ QF.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{(-1)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{-59}}{6}$$

$\rightarrow \therefore$ no sol'ns

Example 3.8.4

Determine the number of points of intersection without solving the System:

$f(x) = x^2 + 2x + 14, \ \ g(x) = 8x + 5$ (Hint: To solve this problem you must be very discriminating)

System: $x^2 + 2x + 14 = 8x + 5$

$\hookrightarrow$ the discriminant $b^2 - 4ac$

$\Rightarrow x^2 - 6x + 9 = 0$

$a = 1 \quad b = -6 \quad c = 9$

$$b^2 - 4ac = (-6)^2 - 4(1)(9)$$

$$= 36 - 36$$

$$= 0$$

$\therefore$ There is one point of intersection

Example 3.8.5 (#9 on Page 199 in your text)

9. Determine the value(s) of k such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$. $\Rightarrow$ No soln's $\Rightarrow$ discriminant is less than zero

System: $-3x^2 - x + 4 = 4x + k$ ($-4x$, $-k$)

$\Rightarrow -3x^2 - 5x + 4 - k = 0$

$a=-3 \quad b=-5 \quad c=(4-k)$

We want discriminant < 0

$b^2 - 4ac < 0$

$(-5)^2 - 4(-3)(4-k) < 0$

$\Rightarrow 25 + 12(4-k) < 0$

$\Rightarrow 25 + 48 - 12k < 0$

$\Rightarrow 73 - 12k < 0$ ($+12k$)

$\Rightarrow 73 < 12k$ ($\div 12$)

$\frac{73}{12} < k$

$\boxed{\Rightarrow k > \frac{73}{12}}$

Example 3.8.6 (#10 in your text)

10. A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, $h(t)$, in metres, t seconds after jumping can be modelled by

$h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and
$h_2(t) = -4t + 142$ after he released his parachute.

How long after jumping did the daredevil release his parachute?

System: $-4.9t^2 + t + 360 = -4t + 142$

$\Rightarrow -4.9t^2 + 5t + 218 = 0$

$a=-4.9 \quad b=5 \quad c=218$

$t = \frac{-5 \pm \sqrt{(5)^2 - 4(-4.9)(218)}}{2(-4.9)} = \frac{-5 \pm \sqrt{4297.8}}{-9.8}$

$t = \frac{-5 + 65.56}{-9.8}$ inadmissible   or $t = \frac{-5 - 65.56}{-9.8} = 7.2$

$\therefore$ After 7.2 seconds the parachute is pulled.

Success Criteria:

- I can solve for the points of intersection by
  1. Making the functions equal to each other
  2. Solving for the zeros (x-coordinates) of the resulting quadratic function
  3. Substituting the zeros into the linear equation to determine the corresponding y-values
- I can identify when solutions are inadmissible