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MATH@TD

Chapter 4 – Exponential Functions

4.2 – Integer Exponents

$2^3 = 2 \times 2 \times 2$

Learning Goal: We are learning to work with integer exponents.

Before beginning, we should quickly review (***ominous music plays***):

THE POWER LAWS

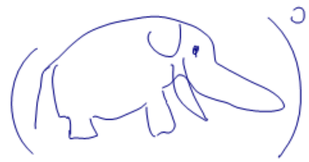
Consider a typical "power" $\boxed{a^n}$. We call "***a***" the BASE. We call "***n***" the EXPONENT and the entire expression a^n is called a POWER

The Laws: Given the powers a^m and a^n, with exponents *m* and *n*, and the number $\frac{a}{b}$, then

1) $1^n = 1$

2) $a^1 = a$

3) $a^0 = 1$

 $^0 = 1$

same base

4) $a^m \cdot a^n = a^{m+n}$

5) $a^m \div a^n = a^{m-n}$

power raised to a new exponent

6) $(a^m)^n = a^{m \cdot n}$

distributive property for powers

7) $(a \cdot b)^m = a^m \cdot b^m$

8) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Until now, for the most part, the exponents you've been working with have always been **Natural Numbers**. But, we now will examine **Integer Exponents**!!

Additional Power Laws:

negative exponent reciprocates (flips) the base

When "flipping" a base change the sign on the exponent

9) $a^{-m} = \left(\frac{1}{a}\right)^{m} = \frac{1^m}{a^m} = \frac{1}{a^m}$

10) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$

11) $\frac{a^{-m}}{b^{-n}} = \frac{b^{+n}}{a^{+m}}$

Example 4.2.1

Write each expression as a single power with a positive exponent:

base to an exponent

÷ power/power w/ same base

a) $(4)^{-5}$

$= \left(\frac{1}{4}\right)^{+5} = \frac{1}{4^5}$

b) $\left(\frac{3}{2}\right)^{-4}$

$= \left(\frac{2}{3}\right)^{+4}$

c) $\frac{7^3}{7^9}$

$= 7^{3-9}$

$= 7^{-6}$

$= \left(\frac{1}{7}\right)^{6} = \frac{1}{7^6}$

$\frac{\cancel{7^3}\ 1}{7^{\cancel{9}\ 6}}$

Example 4.2.2

Simplify, then evaluate each expression and state your answers in rational form:

inside out

fraction

BEDMAS

a) $3^5\left(3^{-2}\right)$

$= 3^{5+(-2)}$

$= 3^3$

$= 27$

b) $\left(2^{-3}\left(2^4\right)\right)^{-5}$

$= \left(2^{-3+4}\right)^{-5}$

$= \left(2^1\right)^{-5}$

$= 2^{-5}$

$= \left(\frac{1}{2}\right)^{5} = \frac{1}{2^5} = \frac{1}{32}$

c) $\frac{5^{-3}}{\left(5^2\right)^{-2}}$

$= \frac{5^{-3}}{5^{2\times(-2)}}$

$= \frac{5^{-3}}{5^{-4}}$

$= 5^{-3-(-4)}$

$= 5^1$

$= 5$

Example 4.2.3

Evaluate and express in rational reduced form:

a) $3^2\left(6^{-3}\right)$

$= \frac{3^2}{6^3} = \frac{9}{216}$

$= \frac{1}{24}$

$= \frac{3^2}{(2\cdot 3)^3}$

$= \frac{\cancel{3^2}\,1}{2^3\cdot \cancel{3^3}\,3} = \frac{1}{24}$

b) $2^{-3}+10^{-3}-3\left(5^{-3}\right)$

$= \frac{1}{8} + \frac{1}{1000} - \frac{3}{125}$

$= \frac{1}{2^3} + \frac{1}{10^3} - \frac{3}{5^3}$

$= \frac{1}{2^3} + \frac{1}{2^3\cdot 5^3} - \frac{3}{5^3}$

$= \frac{5^3}{2^3\cdot 5^3} + \frac{1}{2^3\cdot 5^3} - \frac{3\cdot 2^3}{5^3\cdot 2^3}$

$= \frac{125+1-24}{1000} = \frac{102}{1000}$

$= \frac{51}{500}$

c) $13^{-5}\times\left(\frac{13^2}{13^8}\right)^{-1}$

$= 13^{-5}\times\left(13^{2-8}\right)^{-1}$

$= 13^{-5}\times\left(13^{-6}\right)^{-1}$

$= 13^{-5}\times 13^{+6}$

$= 13^{-5+6} = 13^1$

$= 13$

Example 4.2.4

Evaluate using the laws of exponents (the power rules):

When possible try to get every base the same

a) $3^2\times 9^{-3}\div 3^{-7}$

$9^{-3} = \left(3^2\right)^{-3}$

$= 3^2\times\left(3^2\right)^{-3}\div 3^{-7}$

$= 3^2\times 3^{-6}\div 3^{-7}$

$= 3^{2+(-6)-(-7)}$

$= 3^3 = 27$

b) $\frac{4^{-2}+3^{-1}}{5^{-1}+2^{-2}}$

gross – grunt & work it out

$= \frac{\frac{1}{4^2}+\frac{1}{3}}{\frac{1}{5}+\frac{1}{2^2}}$

$= \frac{\frac{1}{16}+\frac{1}{3}}{\frac{1}{5}+\frac{1}{4}}$

$= \frac{\frac{3+16}{48}}{\frac{4+5}{20}}$

$= \frac{\frac{19}{48}}{\frac{9}{20}}$

$= \frac{19}{\cancel{48}\,12}\times\frac{\cancel{20}\,5}{9}$

$= \frac{95}{108}$

Success Criteria:

- I can apply the exponent laws
- I can recognize that a negative exponent represents a reciprocal expression