Recall the meaning of $5^3 = 5\times5\times5$

multiply "the root" by itself

3 times

$16^{3/4}$

what does it mean to multiply 16 by itself $\frac{3}{4}$ times?

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Chapter 4 – Exponential Functions

4.3 – Rational Exponents

— ratio

Learning Goal: We are learning to work with powers involving rational (fractional) exponents and to evaluate expressions containing them.

A **RATIONAL EXPONENT** can be a **FRACTION**. For example, we can consider the number $(16)^{\frac{3}{4}}$.

radical

no $\sqrt{\ }$

$\sqrt[n]{a} = a^{\frac{1}{n}}$

radicand

$\sqrt{a} = a^{\frac{1}{2}}$

Of course, the question we need to ask is:

Another exponent rule

$a^m = a^n \Rightarrow m = n$

Same base powers = powers

What the rip is that thing??

As you know, a fraction has two parts: a numerator, and a denominator. When a fraction is used as an exponent, the two parts of the fraction carry two related (but different) meanings in terms of "powers".

Recall that 4^3 means $4\times4\times4$. Now $4^{\frac{1}{2}}$ does not mean $4\div4$! Your text has a nice explanation of the meaning of numbers like $4^{\frac{1}{2}}$. See (i.e. **READ** examples 1 and 2 on pages 224 and 225) For now, we will simply take the meaning of

"root"

eg $x^3 = 27$

$\Rightarrow x = (27)^{\frac{1}{3}} = 3$ because $3^3 = 27$

Definition 4.3.1

> Given a power with a "rational" (fractional) exponent $a^{\frac{m}{n}}$, the **numerator** of the exponent is a "**power**" in the usual sense, and the **denominator** represents a "**root**" or "**radical**".

e.g. For the number $16^{\frac{3}{4}}$

Remember $\frac{3}{4} = 3\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)(3)$

separate "exponent part" (numerator) from the "root part" (denominator)

$16^{\frac{3}{4}}$

OR

$= (16^3)^{\frac{1}{4}}$

$= (4096)^{\frac{1}{4}}$

(meaning - there is some #"a" such that $a^4 = 4096$ — 4th root of 4096)

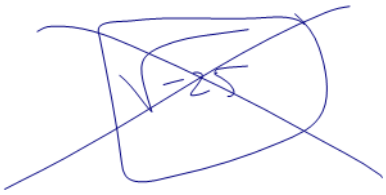
$= (16^{\frac{1}{4}})^3$

$= (2)^3$

$= 8$

It's easier (much) to deal w/ the radical part 1st

$\sqrt[9]{512}$ → reduced form



Example 4.3.1

From your text: Pg. 229 #2.

Write in exponent form, and then evaluate:

a) $\sqrt[9]{512}$

$= (512)^{\frac{1}{9}}$

$= 2$

c) $\sqrt[3]{27^2}$

$= (27^2)^{\frac{1}{3}}$

$= (27^{\frac{1}{3}})^2$

$= (3)^2$

$= 9$

b) $\sqrt[3]{-27}$

$= (-27)^{\frac{1}{3}}$

$= -3.$

$\sqrt{-25}$ = No answer

Note: We ***CANNOT*** take an **Even Root** of a **negative radicand**.
We ***CAN*** take an **Odd Root** of a negative radicand, however.

f) $\sqrt[4]{\left(\frac{16}{81}\right)^{-1}}$

$= \left(\left(\frac{16}{81}\right)^{-1}\right)^{\frac{1}{4}}$

$= \left(\frac{81}{16}\right)^{\frac{1}{4}}$

$= \frac{81^{\frac{1}{4}}}{16^{\frac{1}{4}}}$

$= \frac{3}{2}$

Another

$\left(\left(\frac{16}{81}\right)^{-3}\right)^{\frac{1}{4}}$

$= \left(\left(\frac{81}{16}\right)^{3}\right)^{\frac{1}{4}}$

$= \left(\left(\frac{81}{16}\right)^{\frac{1}{4}}\right)^{3}$

$= \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}.$

Example 4.3.2

From your text: Pg. 229 #3

Write as a single power:

a) $\left(8^{\frac{2}{3}}\right)\left(8^{\frac{1}{3}}\right)$

$= 8^{\frac{2}{3}+\frac{1}{3}}$

$= 8^{\frac{3}{3}}$

$= 8^{1}$

b) $8^{\frac{2}{3}} \div 8^{\frac{1}{3}}$

$= 8^{\frac{2}{3}-\frac{1}{3}}$

$= 8^{\frac{1}{3}}$

Example 4.3.3

From your text: Pg. 229 #4

Write as a single power, then **evaluate**. Express answers in **rational form**. ⇒ fraction

a) $\sqrt{5}\cdot\sqrt{5}$

$= (5)^{\frac{1}{2}} \cdot (5)^{\frac{1}{2}}$

$= 5^{\frac{1}{2}+\frac{1}{2}}$

$= 5^{1}$

$= 5$

b) $\frac{\sqrt[3]{-16}}{\sqrt[3]{2}}$ ← irrational ← irrational

$= \frac{(-16)^{\frac{1}{3}}}{(2)^{\frac{1}{3}}}$

$= \left(\frac{-16}{2}\right)^{\frac{1}{3}}$

$= (-8)^{\frac{1}{3}}$

$= -2$

Success Criteria:

- I can understand that the numerator of a fractional exponent is the power, while the denominator is the root.