Same base - rules 4 & 5

Same exponent - rules 7, 8

A∞Ω
MATH@TD

Chapter 4 – Exponential Functions

4.4 – Simplifying Expressions Involving Exponents

Learning Goal: We are learning to simplify algebraic expressions involving powers and radicals.

Keep the **EXPONENT RULES** in your mind at all times.

One of the Keys of the exponent rules is "**SAMENESS**".

- When you have the **SAME BASE**, (but possibly different exponents) you can combine powers.

e.g. $\dfrac{x^3 \times x^4}{x^7}$ $= x^{3+4-7}$

$= x^0$

$= 1$

- When you have the **SAME EXPONENT** (but possibly different bases) you can "combine the bases under the same exponent".

First, change radical form to exponent form

e.g. $\dfrac{\sqrt[3]{12} \times \sqrt[3]{36}}{\sqrt[3]{16}}$ $= \dfrac{(12)^{\frac{1}{3}} (36)^{\frac{1}{3}}}{(16)^{\frac{1}{3}}}$

$= \left(\dfrac{(\cancel{12}^{3})(\cancel{36}^{9})}{\cancel{4}^{1}\,\cancel{16}}\right)^{\frac{1}{3}} = (3 \cdot 9)^{\frac{1}{3}}$

$= (27)^{\frac{1}{3}} = 3$

$(3 \cdot 3 \cdot 3)^{\frac{1}{3}} = 3$

Now we turn to problems involving both numbers and variables being exponentized (not a word, but it should be because of how awesome it sounds).

Example 4.4.1

Simplify, leaving you answer with only positive exponents:

a) $\left(x^3\right)^2\left(x^{-8}\right)$

$= \left(x^{3\cdot 2}\right)\cdot\left(x^{-8}\right)$

$= \left(x^{6}\right)\cdot\left(x^{-8}\right)$

$= x^{6+(-8)}$

$= x^{-2}$

$= \left(\frac{1}{x}\right)^{+2} = \frac{1}{x^2}$

b) $\dfrac{z^4\left(z^{-2}\right)}{z^{-3}}$

$= z^{4+(-2)-(-3)}$

$= z^{5}$

c) $\sqrt{36x^{-6}}$

$= \left(36x^{-6}\right)^{\frac{1}{2}}$

$= 36^{\frac{1}{2}}\cdot\left(x^{-6}\right)^{\frac{1}{2}}$

$= 6\left(x^{-6\cdot\frac{1}{2}}\right)$

$= 6x^{-3}$ — only affects the x $\Rightarrow$ does not touch the 6

$= \frac{6}{1}\left(\frac{1}{x^3}\right)$

$= \dfrac{6}{x^3}$

d) $\left(\dfrac{\left(3x^2y\right)^{-1}\left(x^3y^{-4}\right)}{\left(x^3y^2\right)^{-2}}\right)^{-2}$ — last

$= \left(\dfrac{\left(x^3y^2\right)^{2}\left(x^3y^{-4}\right)}{\left(3x^2y\right)^{1}}\right)^{-2}$

$= \left(\dfrac{\left(x^6y^4\right)\left(x^3y^{-4}\right)}{3x^2y}\right)$

$= \left(\dfrac{x^{6+3-2}\cdot y^{4+(-4)-1}}{3}\right)^{-2}$

$= \left(\dfrac{x^7\cdot y^{-1}}{3}\right)^{-2}$

e) $\left(\dfrac{\sqrt{16a^6}}{\left(a^3\right)^{-1}}\right)^{\frac{3}{2}}$

$= \left(\dfrac{\left(16a^6\right)^{\frac{1}{2}}}{a^{-3}}\right)^{\frac{3}{2}}$

$= \left(\dfrac{4a^3}{a^{-3}}\right)^{3/2}$

$= \left(4a^{3-(-3)}\right)^{3/2}$

$= \left(4a^6\right)^{3/2}$ → separate the numerator & denominator from the rational exponent

$= \left[\left(4a^6\right)^{\frac{1}{2}}\right]^3$

$= \left(2a^3\right)^3 = 2^3\left(a^3\right)^3$

$= 8a^9$

f) $\left(\dfrac{\left(6x^3\right)^2\left(6y^3\right)}{\left(9xy\right)^6}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{\left(6^2x^6\right)\left(6^1y^3\right)}{9^6x^6y^6}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{6^{2+1}x^{6-6}y^{3-6}}{9^6}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{6^3x^0y^{-3}}{9^6}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{6^3}{9^6y^3}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{9^6y^3}{6^3}\right)^{+\frac{1}{3}}$

$= \dfrac{9^{6\times\frac{1}{3}}y^{3\times\frac{1}{3}}}{6^{3\times\frac{1}{3}}} = \dfrac{9^2y}{6}$

$\dfrac{\cancel{81}^{27}y}{\cancel{6}_2} = \dfrac{27y}{2}.$

$= \left(\dfrac{x^7}{3y}\right)^{-2}$

$= \left(\dfrac{3y}{x^7}\right)^{+2}$

$= \dfrac{3^2 \cdot y^2}{\left(x^7\right)^2}$

$= \dfrac{9y^2}{x^{14}}.$

Success Criteria:

- I can simplify algebraic expressions containing powers by using the exponent laws
- I can simplify algebraic expressions involving radicals