not sinusoidal!!!

Chapter 4 – Exponential Functions

4.5 – 4.6 – Properties and Transformations of Exponential Functions

Learning Goal: We are learning to identify the characteristics and transformations of the graphs and equations of exponential functions.

the exponent is the variable.

Recall: A function is a machine which attaches #'s together to make pairs

Exponential Functions are of the (basic) form:

$$\boxed{f(x) = b^x}$$

(domain, range)

numbers plugged in

numbers which are calculated

(of course, we can apply transformations to this basic, or parent, function!! Fun Times are – a – coming!!)

In the basic exponential function $f(x)=b^x$, b is the base. **THE BASE OF AN EXPONENTIAL FUNCTION IS JUST A NUMBER.** For example, we might have the functions

$f(x) = 2^x$ $\quad$ $g(x) = (1.01)^x$

$h(x) = (0.98)^x$ $\quad$ $m(x) = (5)^x$

What the Base of an Exponential Function tells you

If the base $b > 1$ the function $\boxed{\text{describes}}$ growth

If the base $b = 1$ – the function is the horizontal line $\boxed{y = 1}$

If the base $0 < b < 1$, the function describes decay

Note: "b" is always positive

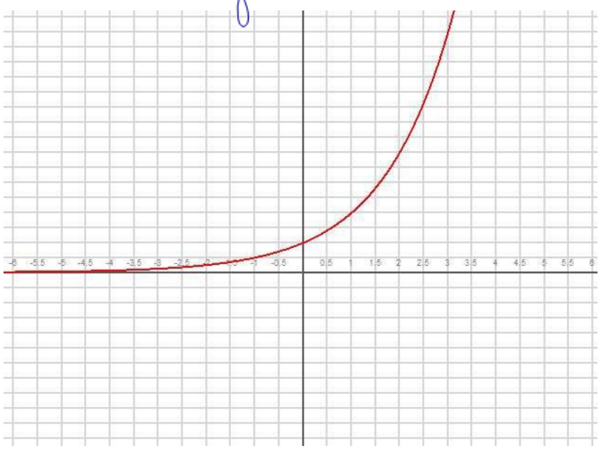
non transformed

Domain and Range of the Basic Exponential Function

$f(x) = b^x$

Consider the sketches:

growth

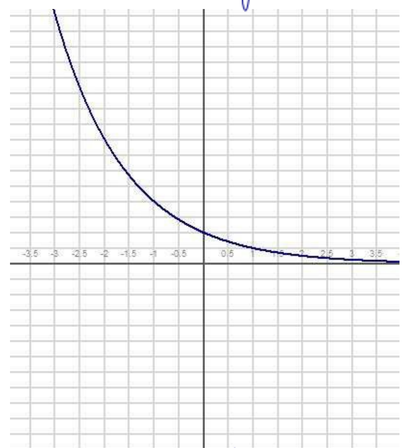


$f(x) = 2^x$

$D: \{x \in \mathbb{R}\}$

$R: \{f(x) \in \mathbb{R} \mid f(x) > 0\}$

decay



$g(x) = \left(\frac{1}{2}\right)^x$

$D: \{x \in \mathbb{R}\}$

$R: \{g(x) \in \mathbb{R} \mid g(x) > 0\}$

ALL Exponential Functions have a Horizontal **ASYMPTOTE** (Basic Exponential

↳ line you cannot cross.

Functions have $y = 0$ as their Horizontal Asymptote. The Horizontal Asymptote of a Transformed Exponential Function depends on

the vertical shift

ALL BASIC Exponential Functions pass through the point $(0,1)$.

$f(x) = b^x$

$(f(0) = b^0 = 1)$

Transformed Exponential Functions will have a y-intercept, but depends on

vertical shift

vertical flip

vertical stretch

horizontal stretch, shift

The Transformed Exponential Function

The general form of an exponential function is:

$$f(x) = a \cdot b^{k(x-d)} + c$$

Where:

a = vertical stretch. If $a<0$ we also have a vertical flip	k = horizontal stretch w/ stretch factor $\frac{1}{k}$. If $k<0$ we also have a horizontal flip
c = vertical shift	d = horizontal shift. Note k must be factored away from x!

Example 4.6.1

State the transformations applied to the parent function $f(x) = 3^x$. Also state the y-intercept, and the equation of the horizontal asymptote of the transformed function.

$$g(x) = -2 \cdot 3^{3x+3} + 4 = -2 \cdot 3^{3(x+1)} + 4$$

	vertical	horizontal
flip	Yes	No.
stretch	-2	$k=3 \Rightarrow$ factor $\frac{1}{3}$
shift	4 up	1 left

y-int: $x = 0$

$g(0) = -2 \cdot 3^{3(0)+3} + 4$

$= (-2)(27) + 4$

$= -50$

Horizontal Asymptote

$y = 4$

$f(x) = a \cdot \left(b^{k(x-d)}\right) + c$

Example 4.6.2

From your Text: Page 252 #7

room temperature (horizontal asymptote)

7. A cup of hot liquid was left to cool in a room whose temperature was 20 °C. The temperature changes with time according to the function $T(t) = 80\left(\frac{1}{2}\right)^{\frac{t}{30}} + 20$. Use your knowledge of transformations to sketch this function. Explain the meaning of the *y*-intercept and the asymptote in the context of this problem.

"time" is being 'chunked' into 30 min blocks

⇒ The temperature is measured every 30 min

$T(60) = 80\left(\frac{1}{2}^{\frac{60}{30}}\right) + 20$

$= 80\left(\frac{1}{2}^{2}\right) + 20$

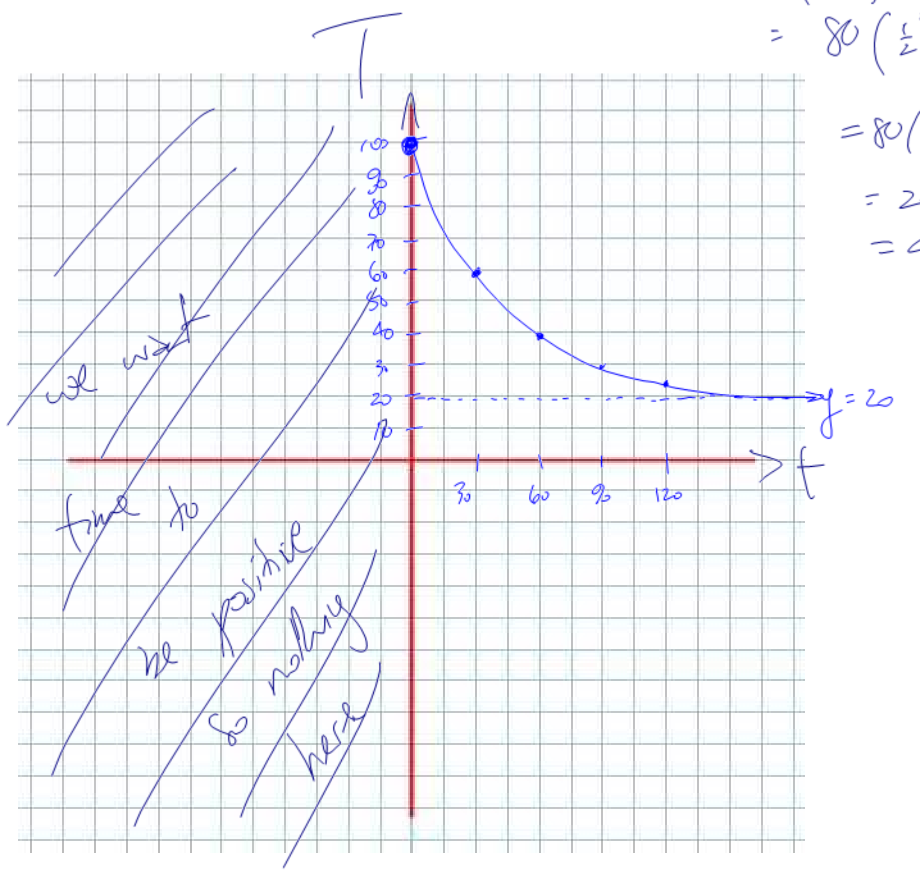
$= 80\left(\frac{1}{4}\right) + 20$

$= 20 + 20$

$= 40$

ToV.

t	$T = 80\left(\frac{1}{2}^{\frac{t}{30}}\right) + 20$
0	100° C
30	60°
60	40°
90	30°
120	25°



$$g(x) = a\left(b^{k(x-d)}\right) + c$$

Example 4.6.3

From your Text: Page 252 #5a

Let $f(x)=4^x$. For the function which follows,

- State the transformations applied to $f(x)$
- State the y-intercept, and the horizontal asymptote
- Sketch the transformed function, and write the function "properly"
- State the domain and range of the transformed function

	vertical	horizontal
flip	No	Yes
stretch	$\frac{1}{2}$	$\times 1$
shift	2 up	No.

$$g(x)=0.5f(-x)\boxed{+2}$$ Horizontal Asymptote

$\frac{1}{2}\left(\frac{1}{16}\right)+2 = \frac{1}{32}+2$

$$g(x)=\frac{1}{2}4^{-x}+2$$

ToV

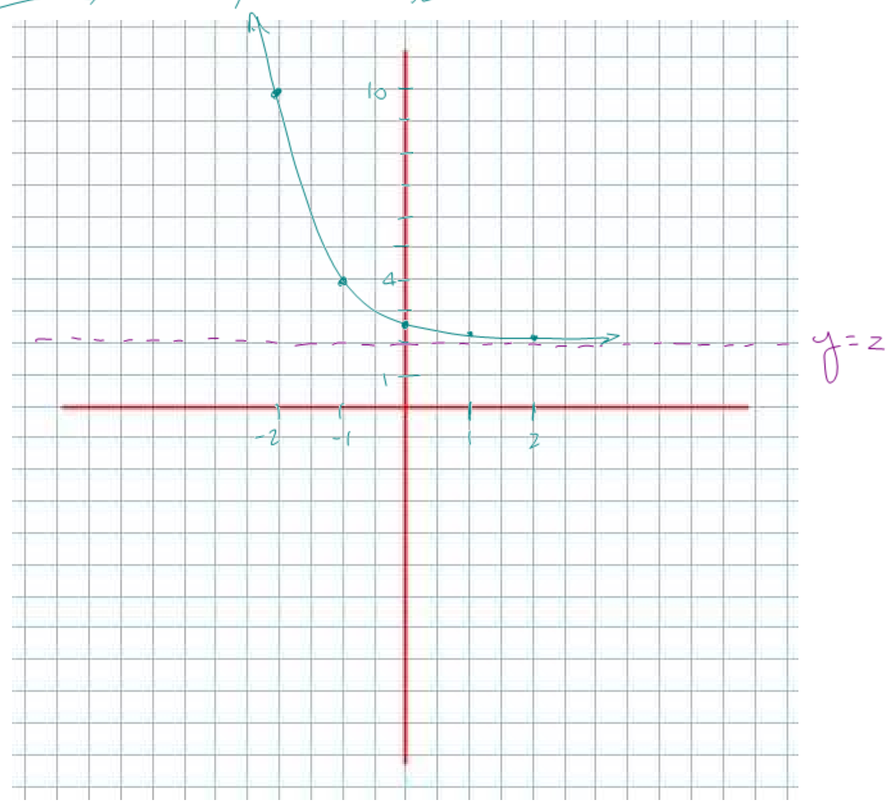
parent

x_p	$f=4^{x_p}$
-2	$\frac{1}{16}$
-1	$\frac{1}{4}$
0	1
1	4
2	16

Transformed

$x_T=-x_p$	$g=\frac{1}{2}f+2$
2	$2\frac{1}{32} = 2.03$
1	2.125
0	2.5
-1	4
-2	10

$4^{-2}=\frac{1}{4^2} = \frac{1}{16}$



Success Criteria:

- I can identify the graph of an exponential function
- I can identify and apply the four transformations (a, k, d, c) to the equation of an exponential function