

# Functions 11

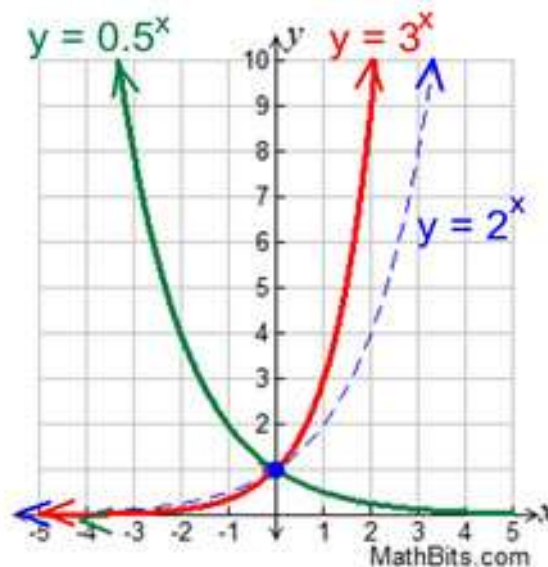
## Course Notes

### Unit 4 – Exponential Functions

#### *GROWTH AND DECAY – WHAT DOES MATH SAY?*

We are learning to

- *know and understand how to use exponent laws to simplify algebraic expressions and to compute values (integer and rational exponents)*
- *sketch graphs of exponential functions*
- *using transformations interpret and sketch exponential functions*
- *solve real-world problems, using algebraic and graphical techniques*



# Chapter 4 – Exponential Functions

*Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check.*

## **Section 4.2**

Pg. 222 – 223 #5 – 8, 13

## **Section 4.3**

Pg. 229: #2de, 3cdef, 4cd, 5, 6, 8, 10 (*a question of awesomeness*), 12 (*we may take up next day*)

## **Section 4.4**

Pg. 236 – 237 #2acef, 4acdf, 5, 6, 7ac (*simplify BEFORE substituting!*), 9ad

## **Section 4.5/4.6**

Pg. 251 – 253 #1 – 3, 5bcd (*write each transformed function “properly”*), 8 – 10 (*for #10 please see example 4 on page 250*)

## **Section 4.7**

**READ** Example 2 on pages 256 – 257 (which method do you prefer: Guess and Check, or Graphing Calculator?)

**READ** Example 4 on pages 259 – 260.

Pg. 261 – 263 #1, 3 – 9, 11 – 16

## Chapter 4 – Exponential Functions

### 4.2 – Integer Exponents

**Learning Goal:** We are learning to work with integer exponents.

Before beginning, we should quickly review (*ominous music plays*):

## THE POWER LAWS

Consider a typical “power”  $a^n$ . We call “ $a$ ” the  $a$ . We call “ $n$ ” the  $n$  and the entire expression  $a^n$  is called a  $a^n$ .

**The Laws:** Given the powers  $a^m$  and  $a^n$ , with exponents  $m$  and  $n$ , and the number  $\frac{a}{b}$ , then

1)

2)

3)

4)

5)

6)

7)

8)

Until now, for the most part, the exponents you've been working with have always been **NATURAL NUMBERS**. But, we now will examine **INTEGER EXPONENTS!!**

***ADDITIONAL POWER LAWS:***

9)

10)

11)

**Example 4.2.1**

Write each expression as a single power with a positive exponent:

a)  $(4)^{-5}$

b)  $\left(\frac{3}{2}\right)^{-4}$

c)  $\frac{7^3}{7^9}$

**Example 4.2.2**

Simplify, then evaluate each expression and state your answers in rational form:

a)  $3^5(3^{-2})$

b)  $(2^{-3}(2^4))^{-5}$

c)  $\frac{5^{-3}}{(5^2)^{-2}}$

**Example 4.2.3**

Evaluate and express in rational form:

a)  $3^2(6^{-3})$

b)  $2^{-3} + 10^{-3} - 3(5^{-3})$

c)  $13^{-5} \times \left(\frac{13^2}{13^8}\right)^{-1}$

**Example 4.2.4**

Evaluate using the laws of exponents (the power rules):

a)  $3^2 \times 9^{-3} \div 3^{-7}$

b)  $\frac{4^{-2} + 3^{-1}}{5^{-1} + 2^{-2}}$

**Success Criteria:**

- I can apply the exponent laws
- I can recognize that a negative exponent represents a reciprocal expression

## Chapter 4 – Exponential Functions

### 4.3 – Rational Exponents

**Learning Goal:** We are learning to work with powers involving rational (fractional) exponents and to evaluate expressions containing them.

A **RATIONAL EXPONENT** can be a **FRACTION**. For example, we can consider the number  $(16)^{\frac{3}{4}}$ .

Of course, the question we need to ask is:

### What the rip is that thing??

As you know, a fraction has two parts: a numerator, and a denominator. When a fraction is used as an exponent, the two parts of the fraction carry two related (but different) meanings in terms of “powers”.

Recall that  $4^3$  **means**  $4 \times 4 \times 4$ . Now  $4^{\frac{1}{2}}$  **does not mean**  $4 \div 4$ ! Your text has a nice explanation of the meaning of numbers like  $4^{\frac{1}{2}}$ . See (i.e. **READ** examples 1 and 2 on pages 224 and 225. For now, we will simply take the meaning of

#### Definition 4.3.1

Given a power with a “rational” (fractional) exponent  $a^{\frac{m}{n}}$ , the **numerator** of the exponent is a “**power**” in the usual sense, and the **denominator** represents a “**root**” or “**radical**”.

e.g. For the number  $16^{\frac{3}{4}}$

**Example 4.3.1**

From your text: Pg. 229 #2.

Write in exponent form, and then evaluate:

a)  $\sqrt[9]{512}$

c)  $\sqrt[3]{27^2}$

b)  $\sqrt[3]{-27}$

f)  $\sqrt[4]{\left(\frac{16}{81}\right)^{-1}}$

Note: We **CANNOT** take an **Even Root** of a **negative radicand**.  
We **CAN** take an **Odd Root** of a negative radicand, however.

**Example 4.3.2**

From your text: Pg. 229 #3

Write as a single power:

a)  $\left(8^{\frac{2}{3}}\right)\left(8^{\frac{1}{3}}\right)$

b)  $8^{\frac{2}{3}} \div 8^{\frac{1}{3}}$

**Example 4.3.3**

From your text: Pg. 229 #4

Write as a single power, then **evaluate**. Express answers in **rational form**.

a)  $\sqrt{5} \cdot \sqrt{5}$

b)  $\frac{\sqrt[3]{-16}}{\sqrt[3]{2}}$

**Success Criteria:**

- I can understand that the numerator of a fractional exponent is the power, while the denominator is the root.

## Chapter 4 – Exponential Functions

### 4.4 – Simplifying Expressions Involving Exponents

**Learning Goal:** We are learning to simplify algebraic expressions involving powers and radicals.

Keep the **EXPONENT RULES** in your mind at all times.

One of the Keys of the exponent rules is “**SAMENESS**”.

- When you have the **SAME BASE**, (but possibly different exponents) you can combine powers.

$$\text{e.g. } \frac{x^3 \times x^4}{x^7}$$

- When you have the **SAME EXPONENT** (but possibly different bases) you can “combine the bases under the same exponent”.

$$\text{e.g. } \frac{\sqrt[3]{12} \times \sqrt[3]{36}}{\sqrt[3]{16}}$$

Now we turn to problems involving both numbers and variables being exponentized (not a word, but it should be because of how awesome it sounds).

**Example 4.4.1**

Simplify, leaving your answer with only positive exponents:

a)  $(x^3)^2(x^{-8})$

b)  $\frac{z^4(z^{-2})}{z^{-3}}$

c)  $\sqrt{36x^{-6}}$

d)  $\left(\frac{(3x^2y)^{-1}(x^3y^{-4})}{(x^3y^2)^{-2}}\right)^{-2}$

$$\text{e) } \left( \frac{\sqrt{16a^6}}{(a^3)^{-1}} \right)^{\frac{3}{2}}$$

$$\text{f) } \left( \frac{(6x^3)^2(6y^3)}{(9xy)^6} \right)^{\frac{1}{3}}$$

**Success Criteria:**

- I can simplify algebraic expressions containing powers by using the exponent laws
- I can simplify algebraic expressions involving radicals

## Chapter 4 – Exponential Functions

### 4.5 – 4.6 – Properties and Transformations of Exponential Functions

**Learning Goal:** We are learning to identify the characteristics and transformations of the graphs and equations of exponential functions.

Exponential Functions are of the (basic) form:

$$f(x) = b^x$$

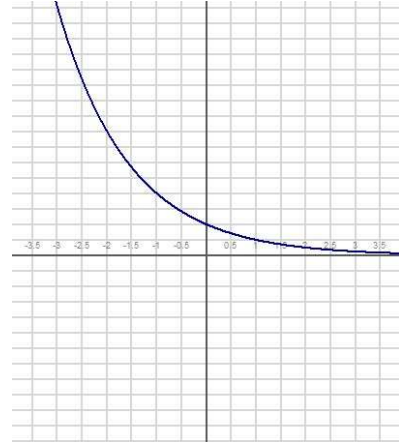
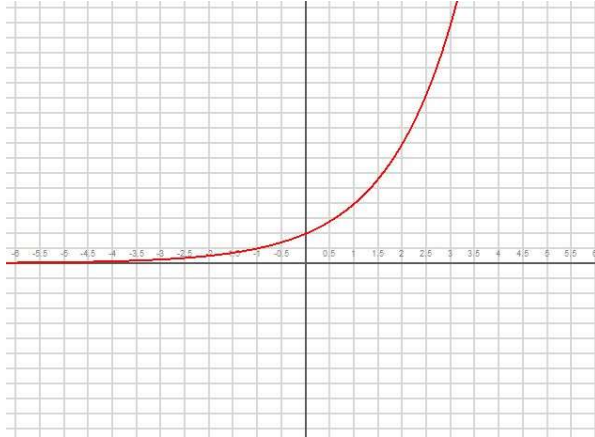
(of course, we can apply transformations to this basic, or parent, function!! Fun Times are – a – coming!!)

In the basic exponential function  $f(x) = b^x$ ,  $b$  is the base. **THE BASE OF AN EXPONENTIAL FUNCTION IS JUST A NUMBER.** For example, we might have the functions

What the Base of an Exponential Function tells you

# Domain and Range of the Basic Exponential Function

Consider the sketches:



**ALL** Exponential Functions have a Horizontal **ASYMPTOTE** (Basic Exponential Functions have  $y = 0$  as their Horizontal Asymptote. The Horizontal Asymptote of a Transformed Exponential Function depends on

**ALL BASIC** Exponential Functions pass through the point **(0,1)**.

Transformed Exponential Functions will have a  $y$ -intercept, but depends on

## The Transformed Exponential Function

The general form of an exponential function is:

$$f(x) = a \cdot b^{k(x-d)} + c$$

Where:

### Example 4.6.1

State the transformations applied to the parent function  $f(x) = 3^x$ . Also state the y-intercept, and the equation of the horizontal asymptote of the transformed function.

$$g(x) = -2 \cdot 3^{3x+3} + 4$$

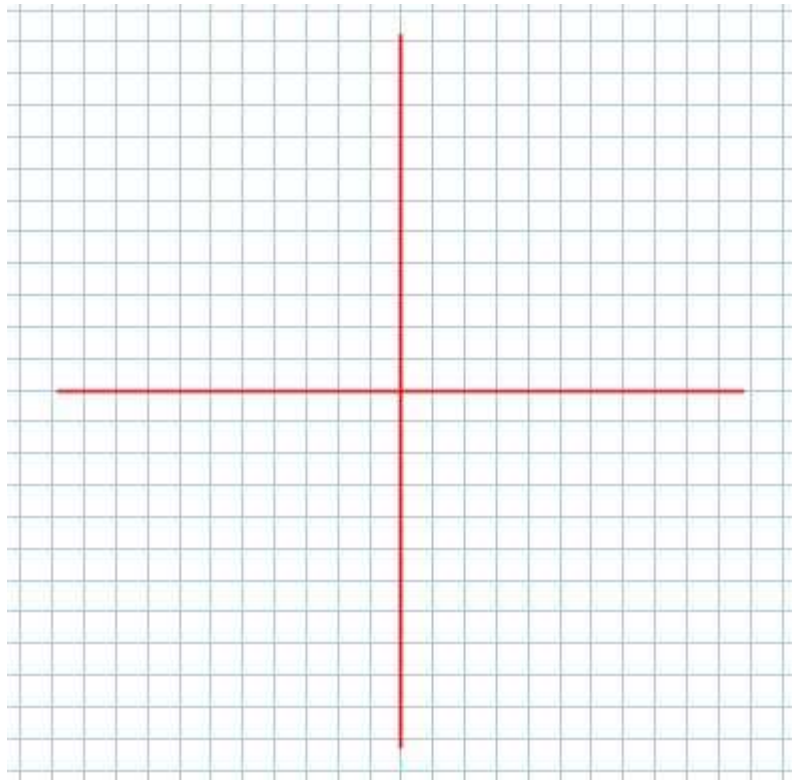
**Example 4.6.2**

From your Text: Page 252 #7

7. A cup of hot liquid was left to cool in a room whose temperature was  $20^\circ\text{C}$ .

**C** The temperature changes with time according to the function

$T(t) = 80\left(\frac{1}{2}\right)^{\frac{t}{30}} + 20$ . Use your knowledge of transformations to sketch this function. Explain the meaning of the  $y$ -intercept and the asymptote in the context of this problem.



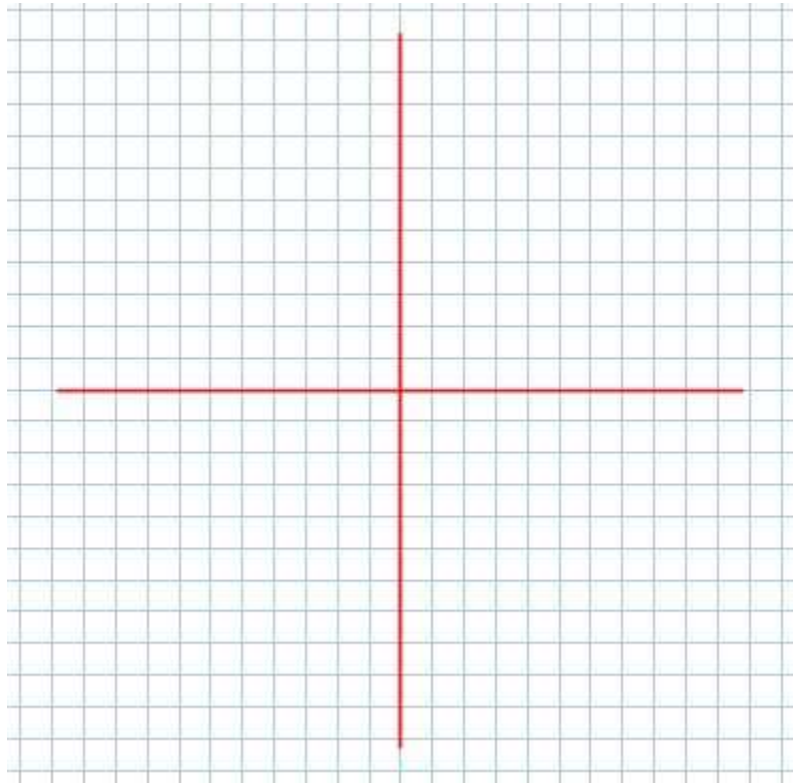
### Example 4.6.3

From your Text: Page 252 #5a

Let  $f(x) = 4^x$ . For the function which follows,

- State the transformations applied to  $f(x)$
- State the y-intercept, and the horizontal asymptote
- Sketch the transformed function, and write the function “properly”
- State the domain and range of the transformed function

$$g(x) = 0.5f(-x) + 2$$



### Success Criteria:

- I can identify the graph of an exponential function
- I can identify and apply the four transformations (a, k, d, c) to the equation of an exponential function

## Chapter 4 – Exponential Functions

### 4.7 – Applications of Exponential Functions

**Learning Goal:** We are learning to use exponential functions to solve problems involving exponential growth and decay.

Anything in the real world which grows, or decays can be “**MODELED**” (or in some sense “**DESCRIBED**”) with words, or pictures or **mathematics**. **Mathematical models** are useful for getting solutions to problems, and making predictions.

So far in Mathematics 11U we have studied the basics of functions in general (chapter 1), we’ve done some algebra (chapter 2), and we’ve examined Quadratic functions (chapter 3). Part of our study of Quadratics was learning how to use the vertex of a parabola to answer questions about maxima and minima for some real word problems. For example we saw a question where we tried to maximize revenue for a school store. Quadratic **MODELS** are very useful for solving max/min problems.

In this lesson we want to work on **LEARNING HOW TO SOLVE PROBLEMS DEALING WITH GROWTH AND DECAY**. We have to decide what type of function will best model (or describe) the type of growth/decay seen in the problem (hint: for this lesson we’ll be examining Exponential Growth and Decay, and therefore we expect that exponential functions will be used...shocking, I know)

#### Q. What is Exponential Growth or Decay?

Consider the following:

A single cell divides into two “daughter” cells. Both daughter cells divide resulting in four cells. Those four cells each divide and we now have a population of

Describe, using mathematics, how the cell population changes from generation to generation.

**Example 4.7.1**

Being a financial wizard, you deposit \$1,000 into an account which pays 3.5% interest, annually.

- a) Determine how much money is in your account after  $t = 1, 2, 3,$  and 4 years.
- b) Determine a mathematical model which can describe how the value of the account is changing from year to year.

**Definition 4.7.1**

A function describing Exponential Growth is of the form:

A function describing Exponential Decay is of the form:





### **Example (Half-Life)**

A 200g sample of radioactive material has a half-life of 138 days. How much will be left in 5 years?

### **Success Criteria:**

- I can differentiate between exponential growth and exponential decay
- I can use the exponential function  $f(x) = ab^x$  to model and solve problems involving exponential growth and decay
  - Growth rate is  $b = 1 + r$ . Decay rate is  $b = 1 - r$ .
  - $r$  is a DECIMAL, not a percent!!!!