Unit 5 – Trigonometric Ratios

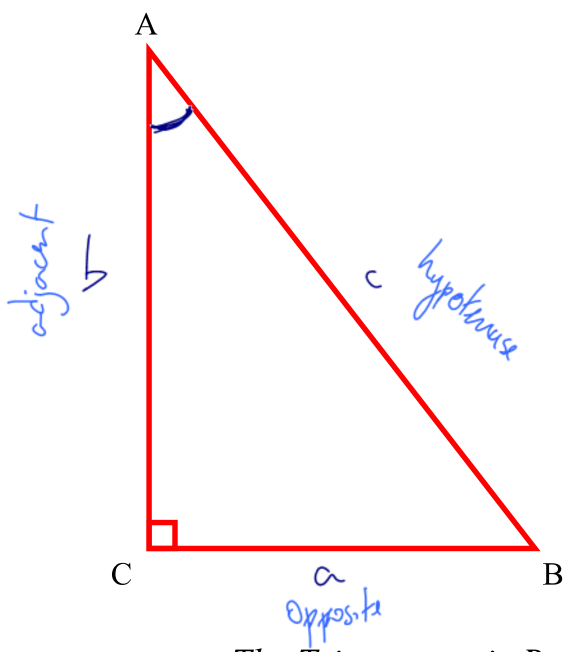
5.1 – Trigonometric Ratios of Acute Angles

Learning Goal: We are learning to evaluate trigonometric ratios and their reciprocals.

Recall from Grade 10 the mnemonic

SOH CAH TOA

We use SOH CAH TOA to calculate the so-called "trig ratios" for a **right angle triangle**.
Consider the triangle:



focus on angle A ($\measuredangle A$)

The Trigonometric Ratios

Primary Trig Ratios – "fraction" "number"

Reciprocal Trig Ratios

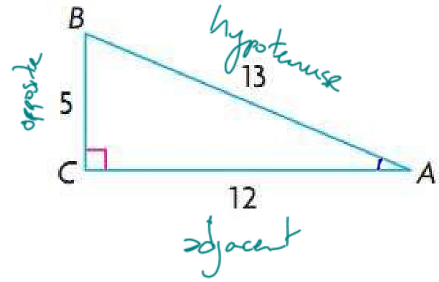
sine: $\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$ | $\frac{1}{\text{sine}} = \text{cosecant} = \csc(A) = \frac{c}{a}$

cosine: $\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$ | $\frac{1}{\text{cosine}} = \text{secant} = \sec(A) = \frac{c}{b}$

tangent: $\tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$ | $\frac{1}{\text{tangent}} = \text{cotangent} = \cot(A) = \frac{b}{a}$

Example 5.1.1

From your text, Pg. 280 #1 Given $\triangle ABC$, state the six trigonometric ratios for $\angle A$.



$\sin(A) = \frac{5}{13}$ $\quad$ $\csc(A) = \frac{13}{5}$

$\cos(A) = \frac{12}{13}$ $\quad$ $\sec(A) = \frac{13}{12}$

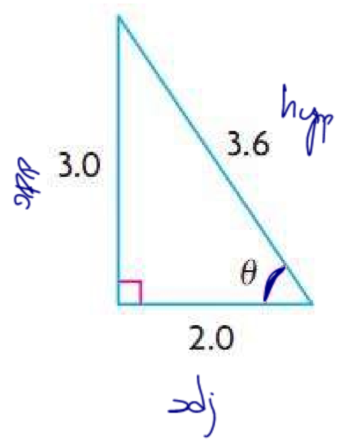
$\tan(A) = \frac{5}{12}$ $\quad$ $\cot(A) = \frac{12}{5}$

Example 5.1.2

For the given right triangle determine:

a) $\csc(\theta)$, $\sec(\theta)$, and $\cot(\theta)$.

b) the angle θ to the nearest degree.



theta

$\frac{hyp}{opp}$

a) $\csc(\theta) = \frac{3.6}{3.0} = 1.2$

$\sec(\theta) = \frac{3.6}{2.0} = 1.8$

hyp / adj

$\cot(\theta) = \frac{2.0}{3.0} = \frac{2}{3} = 0.\dot{6}$

b) $\csc(\theta) = 1.2$

$\Rightarrow \theta = \csc^{-1}(1.2)$

$\doteq 56^\circ$

Note: many calculators have no "csc", "sec", "cot" buttons

$\Rightarrow$ convert reciprocal trig ratios to primary

$\csc(\theta) = 1.2$

$\frac{1}{\sin(\theta)} = 1.2$

$\Rightarrow \sin(\theta) = \frac{1}{1.2}$

$\therefore \theta = \sin^{-1}\left(\frac{1}{1.2}\right)$

$\doteq 56^\circ$

Example 5.1.3

a) Determine the corresponding reciprocal ratio:

i) $\sin(\theta) = \frac{2}{5}$ $\qquad$ ii) $\tan(\theta) = -3$

$\csc(\theta) = \frac{5}{2}$ $\qquad$ $\cot(\theta) = -\frac{1}{3}$

b) Calculate to the nearest hundredth: $\sec(34^\circ) = \frac{1}{\cos(34)} \doteq 1.21$

$\sec(34) \doteq 1.21$ $\;\|\;$ or convert to primary

c) Determine the value of θ to the nearest degree: $\csc(\theta) = 2.46$

$\csc(\theta) = 2.46$

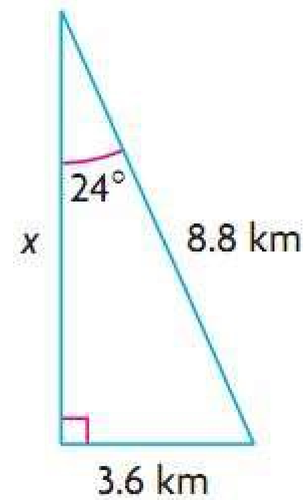
$\Rightarrow \theta = \csc^{-1}(2.46) \doteq 24^\circ$

or $\sin(\theta) = \frac{1}{2.46}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{2.46}\right) \doteq 24^\circ$

Example 5.1.4

Given the right triangle, determine the unknown side using two different trig ratios:



Given info

24° angle

x – unknown | adjacent

3.6 km – opposite

8.8 – hypotenuse

tangent

$\tan(24) = \frac{3.6}{x}$

$x \cdot \tan(24) = 3.6$

$\Rightarrow x = \frac{3.6}{\tan(24)}$

$\doteq 8.09$

$(\doteq 8.1)$

Cosine

$\cos(24) = \frac{x}{8.8}$

$\Rightarrow x = 8.8 \cdot \cos(24)$

$\doteq 8.04$

$(\doteq 8.0)$

the "difference" is due to some bad rounding on the triangle side

Example 5.1.5

From your text, Pg. 282 #11

A kite is flying 8.6 m above the ground at an angle of elevation of 41°. Calculate the length of string, to the nearest tenth of a metre, needed to fly the kite using

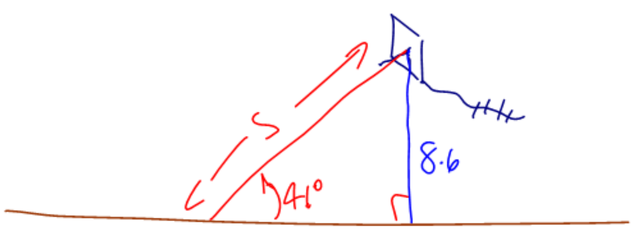
a) a primary trigonometric ratio

b) ~~a reciprocal trigonometric ratio~~

Sine, cosine or tangent ?

opposite/hypotenuse!

Picture



$\sin(41) = \frac{8.6}{S}$

$\Rightarrow S \cdot \sin(41) = 8.6$

$\Rightarrow S = \frac{8.6}{\sin(41)} \doteq 13.1 \text{ m.}$

$\therefore$ the string is 13.1 m.

Success Criteria:

- I can use SohCahToa to determine the primary and reciprocal trigonometric ratios
- I can evaluate problems using the reciprocal trigonometric ratios