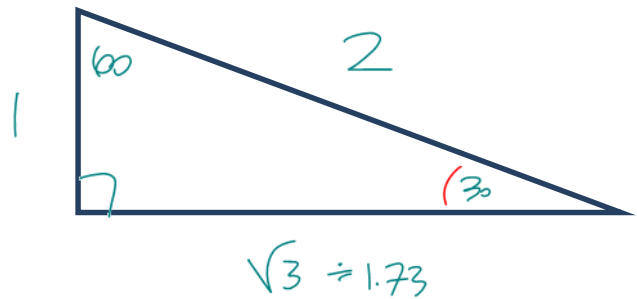
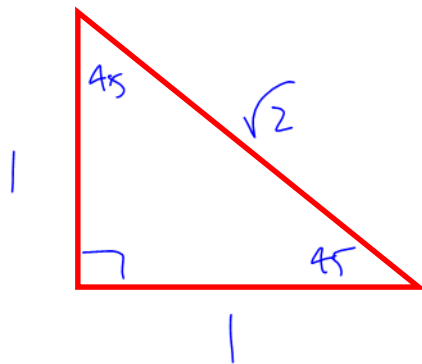


## Unit 5 – Trigonometric Ratios

### 5.2 – Trigonometric Ratios and Special Triangles

**Learning Goal:** We are learning to find the EXACT values of sin, cos, and tan for specific angles.

There are two “**Special Triangles**”



**MEMORIZE THESE!**

SOH CAH TOA

The Primary Trigonometric Ratios of the Special Angles

$$\sin(30^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \sin(45^\circ) &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\begin{aligned} \cos(45^\circ) &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

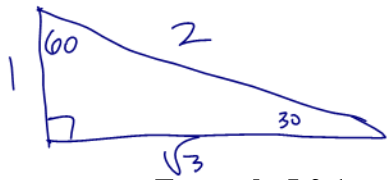
$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\tan(60^\circ) = \frac{\sqrt{3}}{1}$$

$$\tan(45^\circ) = \frac{1}{1} = 1$$

Converting to a rational denominator  
(rationalizing the denominator)

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



**Example 5.2.1**

Evaluate **exactly**

a)  $\sin(45) \cdot \cos(60)$

$$= \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{2}\right)$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4}$$

c)  $\tan(60) \cdot \cos(60) - \sin(60)$

$$= \left(\frac{\sqrt{3}}{1}\right) \left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

$$= 0$$

$$\left. \begin{aligned} 2\sqrt{2} \cdot \sqrt{2} \\ (2)(2) \\ = 4 \end{aligned} \right\}$$

**Example 5.2.2**

Determine the angle  $\theta$  (where  $0 \leq \theta \leq 90^\circ$ ) given:

a)  $\sec(\theta) = \frac{2}{\sqrt{3}}$

$$\cos(\theta) = \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ$$

b)  $\tan(\theta) = \frac{\sqrt{3}}{3}$

$$\tan(\theta) = \frac{1}{\sqrt{3}}$$

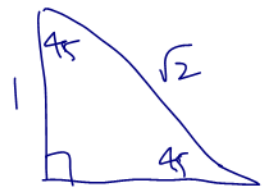
$$\theta = 30^\circ$$

$$= \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\left(\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}\right)$$

No calculator except for the special  $\Delta$ 's



b)  $\cos^2(30) + \sin^2(30)$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{3}{4}$$

d)  $\tan(30) \cdot \frac{\sin(60)}{\cos(45)}$

$$\left(\frac{1}{\sqrt{3}}\right) \cdot \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$$

$$= \left(\frac{1}{\sqrt{3}}\right) \cdot \left(\frac{\sqrt{3}}{2}\right) \cdot \left(\frac{\sqrt{2}}{1}\right) = \frac{\sqrt{6}}{2\sqrt{3}}$$

**Success Criteria:**

- I can draw the two special triangles
- I can identify the EXACT values for  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , using the special triangles
- I can evaluate EXACTLY (no calculators...OR capes!!!) problems involving the special triangles

$$\Rightarrow \sin(\theta) = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$