Unit 5 – Trigonometric Ratios

5.3 – 5.4 – Trigonometric Ratios for Angles Larger than 90º

Learning Goal: We are learning to use a Cartesian plane to evaluate trig ratios for angles between 0º and 360º.

Angles Larger than 90º

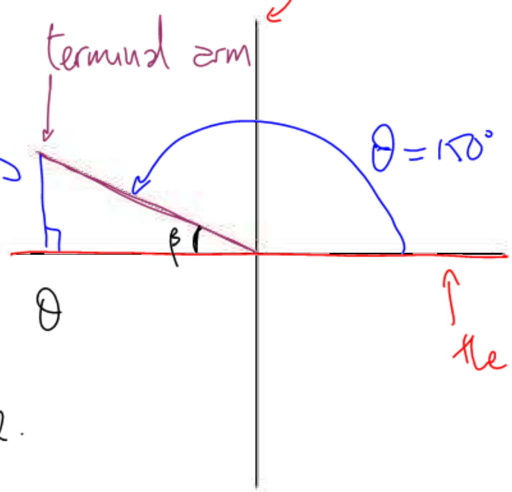
angle of rotation

start on the positive pole and rotate counter clock-wise to a terminal arm

Consider the following sketch of the angle $\theta = 150^\circ$

perpendicular dropped to the pole

vertical axis



e.g. Calculate

$\sin(150) = 0.5$

$\sin(30) = 0.5$

$\cos(150) = -0.866$

$\cos(30) = +0.866$

$\tan(150) = -0.577$

$\tan(30) = +0.577$

β is related to θ through the pole.

In $\boxed{Q2}$ $\beta = 180 - \theta$

β is between the pole and the terminal arm NOT the vertical axis.

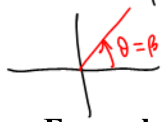
In this example, we call $\theta = 150^\circ$ the **PRINCIPAL ANGLE**, or angle in standard position.

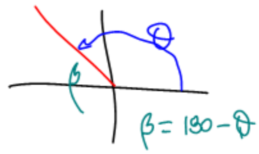
Note: The angle $\beta = 30^\circ$ is called the

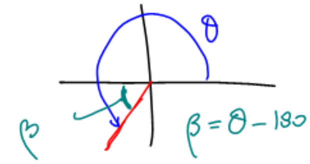
RELATED ACUTE ANGLE

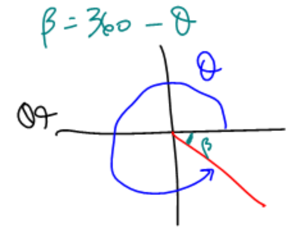
related to the angle of rotation through the pole!

Quadrants and β and θ





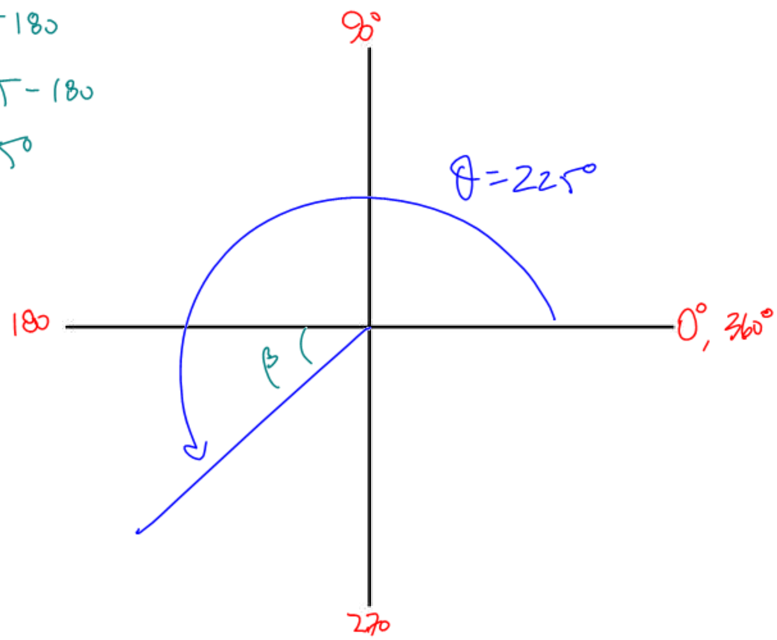




Example 5.3.1

Sketch the angle of rotation $\theta = 225^\circ$ and determine the related acute angle.

$\beta = \theta - 180$
$= 225 - 180$
$= 45°$



e.g. Calculate

$\sin(225) = -0.707$
$\sin(45) = +0.707$
$\cos(225) = -0.707$
$\cos(45) = +0.707$
$\tan(225) = +1$
$\tan(45) = +1$

What is up with these signs??? (**Be Careful with your signs!!!!!!!!!**)

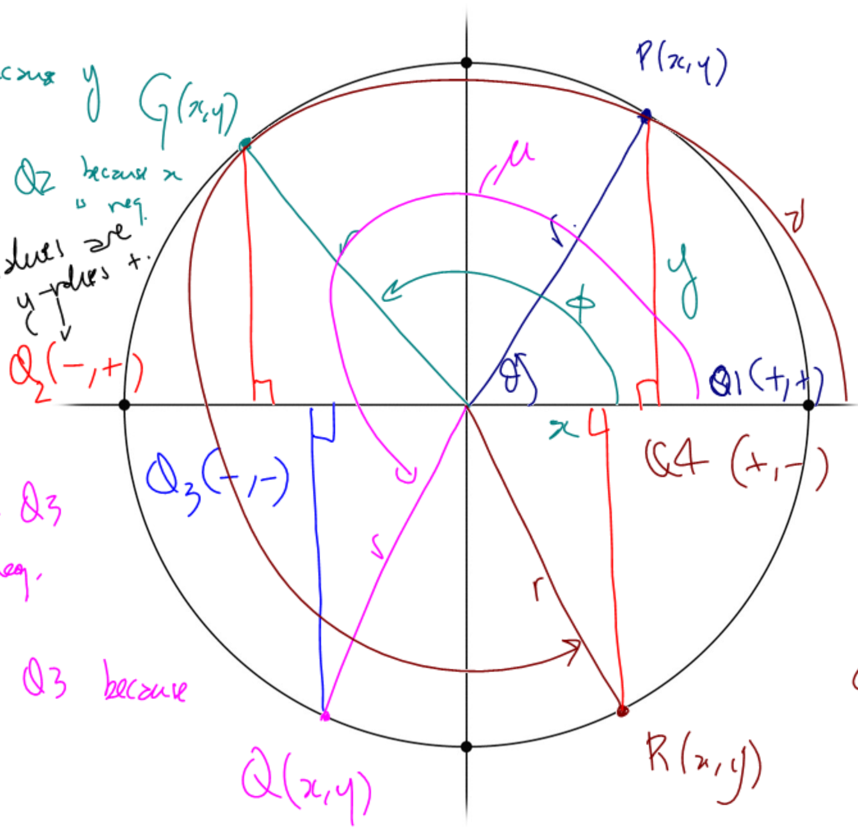
ϕ = phi
μ = mu (mew)
ν = nu

Looking at the TRIG ratios on a Cartesian Plane

sin(φ) is positive because y is +ve

cos(φ) is negative in Q2 because x is neg.

tan(φ) is negative in Q2 because x is neg. x-values are neg. y-values are +.



$\sin(\theta) = \frac{y}{r}$

$\cos(\theta) = \frac{x}{r}$

$\tan(\theta) = \frac{y}{x}$

sin(μ) is neg in Q3 because y is neg.

cos(μ) is neg in Q3 because x is neg.

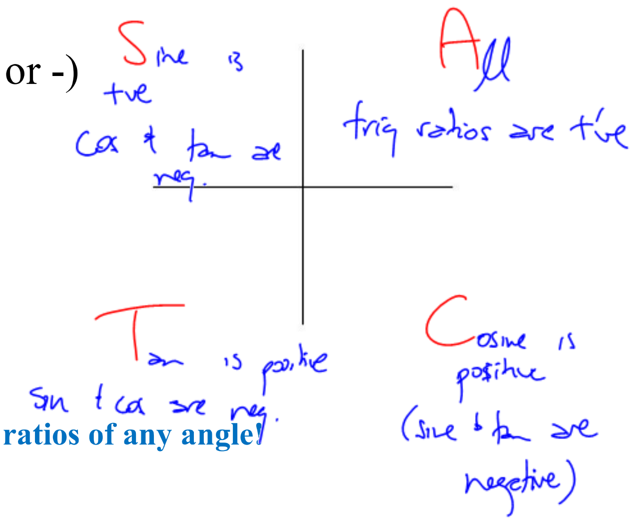
tan(μ) = is positive in Q3

sin(ν) is negative (y is neg.)

cos(ν) is positive (x is positive)

tan(ν) is negative because y is neg. and x is positive

The **CAST RULE** determines the sign (+ or -) of the trig ratio



We now have enough tools to calculate the trigonometric ratios of any angle!

For any given angle θ we will:

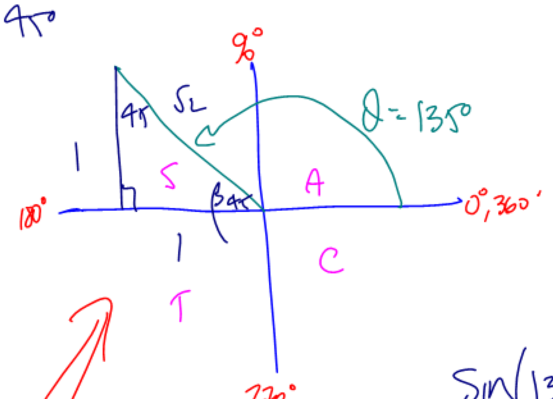
1) Draw θ in **STANDARD POSITION** (i.e. draw the principal angle for θ)
2) Determine the **RELATED ACUTE ANGLE** (β) (between the terminal arm and the x-axis (also called the polar axis)
3) Use the related acute angle and the **CAST RULE** (and SOH CAH TOA) to determine the trig ratio (along with its sign…BE CAREFUL WITH YOUR SIGNS) in question

and special Δs.

Example 5.3.2

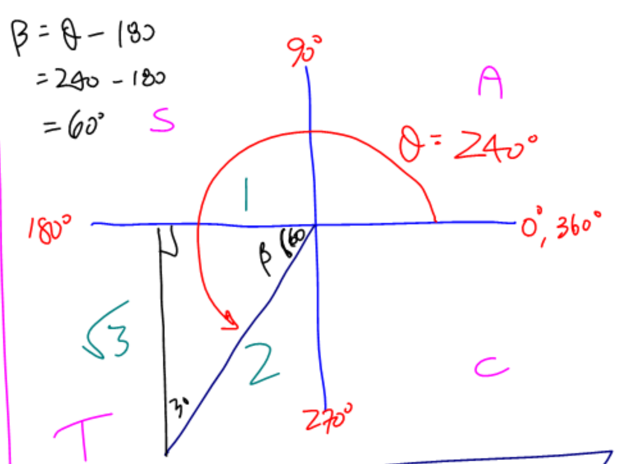
$\beta = 180 - 135$
$= 45°$

Determine the trig ratio $\sin(135)$



$\sin(135) = +\frac{1}{\sqrt{2}}$

Determine the trig ratio ~~tan~~ cos (240)



$\cos(240) = -\frac{1}{2}$

① the special Δ gives the RATIO

② CAST gives the sign on that ratio

Example 5.3.3

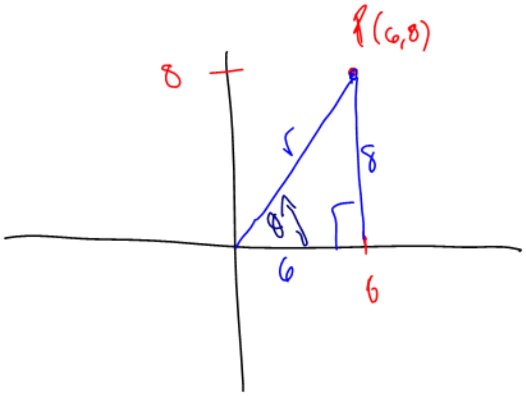
The point $P(x, y) = (6, 8)$ lies on the terminal arm (of length r) of an angle of rotation.

Sketch the angle of rotation.

Determine: a) the value of r

b) the primary trig ratios for the angle

c) the value of the angle of rotation in degrees, to two decimal places



a) $r^2 = 6^2 + 8^2$

$r^2 = 36 + 64$

$r^2 = 100$

$r = 10$

b) $\sin(\theta) = \frac{8}{10} = \frac{4}{5}$

$\cos(\theta) = \frac{6}{10} = \frac{3}{5}$

$\tan(\theta) = \frac{8}{6} = \frac{4}{3}$

c) $\cos(\theta) = \frac{3}{5}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{3}{5}\right) \doteq 53.13^\circ$

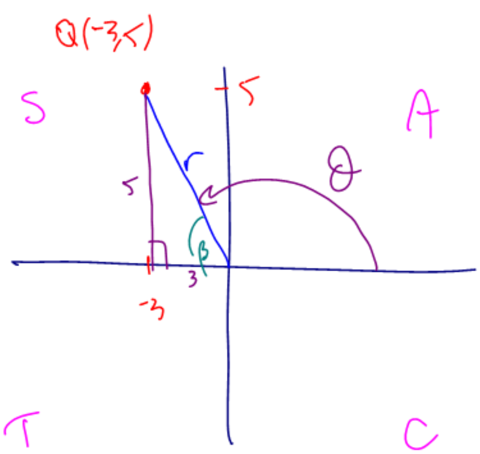
Example 5.3.4

The point $(-3, 5)$ lies on the terminal arm (of length r) of an angle of rotation. Sketch the angle of rotation.

Determine: a) the value of r

b) the primary trig ratios for the angle

c) the value of the angle of rotation in degrees, to two decimal places



a) $r^2 = 3^2 + 5^2$

$r^2 = 9 + 25$

$r^2 = 34$

$r = \sqrt{34}$

$(\doteq 5.83)$

b) $\sin(\theta) = +\frac{5}{\sqrt{34}}$

$\cos(\theta) = -\frac{3}{\sqrt{34}}$

$\tan(\theta) = -\frac{5}{3}$

c) WAY MORE DIFFICULT.

For angles of rotation in Q2, Q3, or Q4, ignore signs on the ratios and find β, then find θ

If $\cos(\theta) = -\frac{3}{\sqrt{34}}$, then

$\cos(\beta) = +\frac{3}{\sqrt{34}} \Rightarrow \beta = \cos^{-1}\left(\frac{3}{\sqrt{34}}\right)$

$\doteq 59^\circ$

$\therefore \theta = 180 - \beta = 180 - 59 = \underline{121^\circ}$

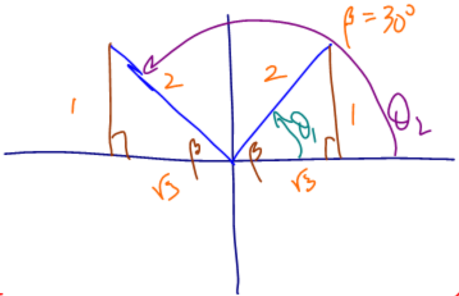
special △'s

Q1, Q2

$\theta = \sin^{-1}(0.5) = 30^\circ$

Example 5.3.5 (going backwards!)

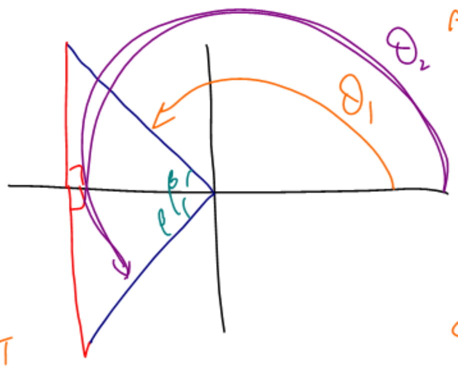
a) Given $\sin(\theta) = +\frac{1}{2}$ determine BOTH values of θ for $0^\circ \le \theta \le 360^\circ$



$\theta_1 = \beta = 30^\circ$

$\theta_2 = 180 - \beta$
$= 180 - 30$
$= 150^\circ$

b) Given $\cos(\theta) = -0.5372$ determine BOTH values of θ for $0^\circ \le \theta \le 360^\circ$



$\cos(\beta) = +0.5372$

$\beta = \cos^{-1}(0.5372)$

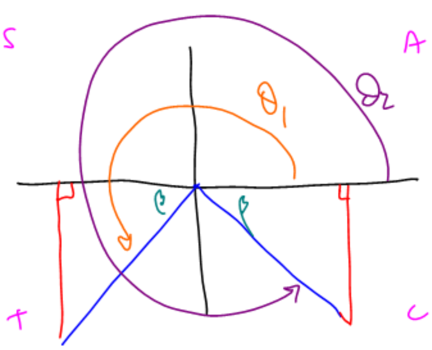
$\doteq 58^\circ$

$\theta_1 = 180 - \beta$
$= 180 - 58$
$= 122^\circ$

$\theta_2 = 180 + \beta$
$= 180 + 58$
$= 238^\circ$.

c) Given $\sin(\theta) = -0.4567$ determine BOTH values of θ for $0^\circ \le \theta \le 360^\circ$

sine is neg. in Q3, Q4



$\sin(\beta) = +0.4567$

$\Rightarrow \beta = \sin^{-1}(0.4567)$

$\doteq 27^\circ$

$\theta_1 = 180 + \beta$
$= 180 + 27$
$= 207^\circ$

$\theta_2 = 360 - \beta$
$= 360 - 27 = 333^\circ$.

Success Criteria:

- I can identify a positive or negative angle based on the direction of rotation
- I can identify the related acute angle (ø) if the principal angle (Θ) lies in quadrants 2, 3, or 4
- I can identify where a trigonometric ratio is + or – using the CAST Rule
- I can recognize that (except for axis angles) every trigonometric ratio has two principal angles between 0º and 360º