

## Unit 5 – Trigonometric Ratios

### 5.6: The Sine Law

**Learning Goal:** We are learning to use the Sine law to solve non-right angle triangles.

Last year you learned the Sine Law. It is a “formula” we can use to **solve triangles which are not right angle triangles**. There is one requirement to be able to use the Sine Law.

**You Must Have an Angle With Its Corresponding Side!**

So far we have been using Right Angle Triangles along with SOH CAH TOA to “solve” triangles. BUT right angle triangles aren’t always the best triangle to use;

Sometime using a right angle triangle just can’t be done. We then need to use so-called “**OBLIQUE TRIANGLES**”. Oblique triangles come in two forms:

- 1) Acute (all angles are less than  $90^\circ$ )
- 2) Obtuse (one angle is more than  $90^\circ$ )

### The Sine Law *(for oblique triangles)*

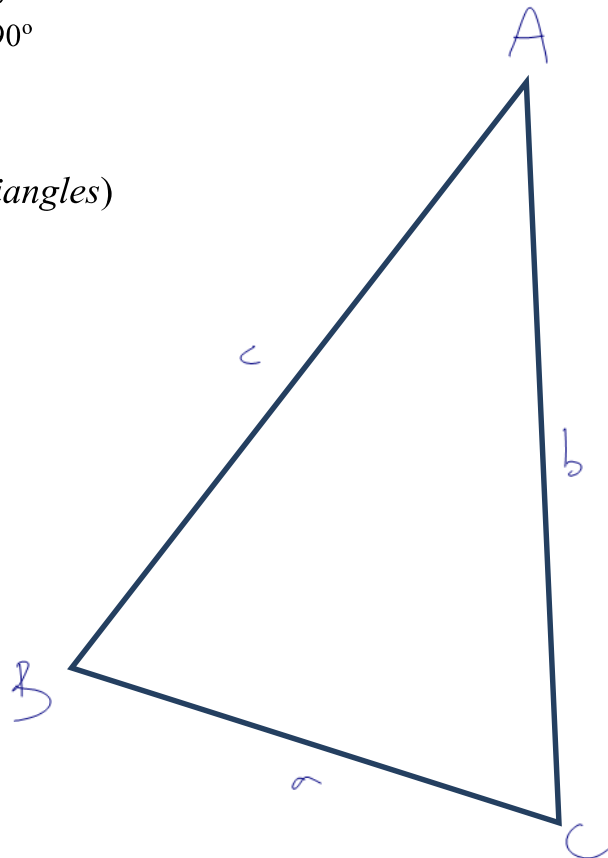
(There are **TWO FORMS** you should know!!)

Given the non-right triangle,  $\triangle ABC$ , then:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

or

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$



**Notes:**

- 1) Memorize the SINE LAW!
- 2) If we are trying to **find an angle**, use the first form of the Sine Law (**angles on top**)
- 3) If we are trying to **find the length** of a side, use the second form of the law (**with sides on top**)
- 4) In order to use the Sine Law, you must have the correct information in the triangle. You must have:
  - a) 3 pieces of information
  - b) One **“CORRESPONDING PAIR”** – an angle with its opposite side (for example you might have side  $a$  and angle  $A$ )

Note: **There is a problem with the Sine Law**

Recall that for trig ratios, “sine” is positive in quadrants 1 **and** 2.

e.g.  $\sin(51^\circ) = 0.777$

$\sin(129^\circ) = 0.777$

What if after a bunch of work we have to calculate

$\theta = \sin^{-1}(0.777)?$

Your calculator gives **ONLY** the acute angle

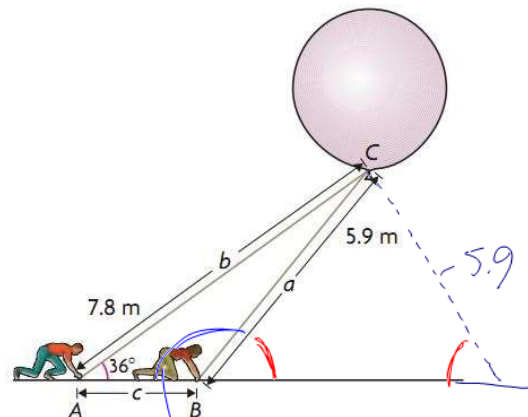
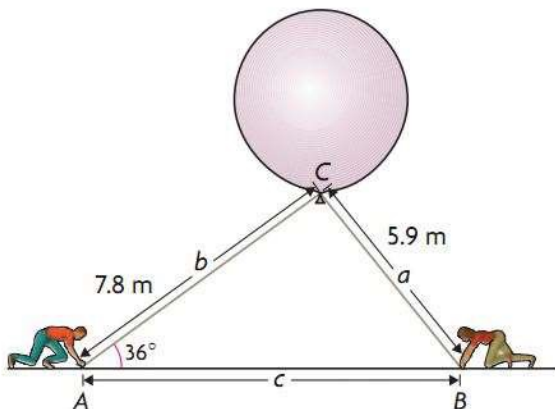
⇒ we have to deal w/ the obtuse.

Consider Example 1 in your text: Pg. 312 – 314 .

Albert and Belle are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of a cloud. Albert's rope is 7.8 m long and makes an angle of  $36^\circ$  with the ground. Belle's rope is 5.9 m long.

The question we are asked is: **How far is Albert from Belle, to the nearest meter?**

Possible Pictures:



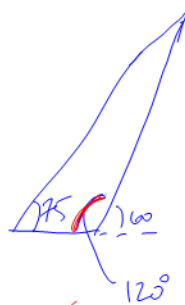
180 - " " } calculator gives this

Both pictures describe the problem completely. So which is correct? Well...BOTH ARE POSSIBLE solutions. This is known as the "AMBIGUOUS CASE". Because both are possible solutions, you must find both.

(BUM CASE)

Note: If the GIVEN ANGLE is ACUTE, then this so called Ambiguous Case MAY APPLY. But, if the Given Angle is Obtuse, then the Ambiguous Case CANNOT APPLY. (And Sometimes, there is no triangle which solves the problem.)

Why?



impossible Δ.

No BUM CASE

given an acute angle → possible BUM case

**Example 5.6.1**

Solve the triangles above.

SL = sine law

$$\angle A = 36^\circ \text{ (g)} \quad a = 5.9 \text{ (g)}$$

$$\angle B = 51^\circ \text{ (SL)} \quad b = 7.8 \text{ (g)}$$

$$\angle C = 93^\circ \text{ (ASTT)} \quad c = 10 \text{ (SL)}$$

ASTT = angle-sum-triangle-theorem

$$\frac{\sin B}{b} = \frac{\sin A}{a} \quad \text{or } (+180)$$

$$\frac{\sin(B)}{7.8} = \frac{\sin(36)}{5.9}$$

$$\sin(B) = \frac{(7.8)(\sin(36))}{5.9}$$

$$\therefore B = \sin^{-1}\left(\frac{(7.8)(\sin(36))}{5.9}\right)$$

$$\hat{=} 51^\circ$$

$$\angle C = 180 - 51 - 36 = 93^\circ$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin(93)} = \frac{5.9}{\sin(36)}$$

$$\Rightarrow c = \frac{(5.9)(\sin(93))}{\sin(36)}$$

$$\hat{=} 10 \text{ m}$$

∴ Albert + Belle are about 10 m apart

BUM CASE

$$180 - 51$$



$$\angle A = 36^\circ \text{ (g)} \quad a = 5.9 \text{ (g)}$$

$$\angle B = 129^\circ \text{ (SAT)} \quad b = 7.8 \text{ (g)}$$

$$\angle C = 15^\circ \text{ (ASTT)} \quad c =$$

$$\frac{c}{\sin(C)} = \frac{a}{\sin(A)}$$

$$\frac{c}{\sin(15)} = \frac{5.9}{\sin(36)}$$

$$c = \frac{(5.9)(\sin(15))}{\sin(36)}$$

$$\hat{=} 2.6 \text{ m}$$

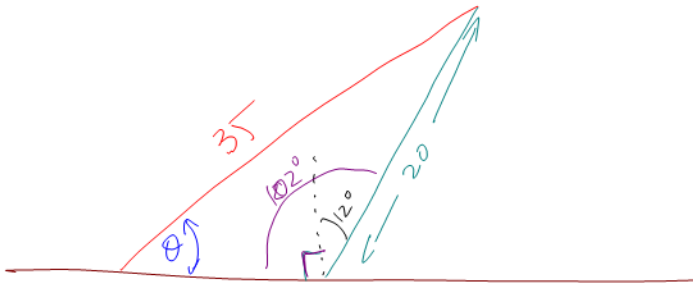
∴ Albert + Belle are 2.6 m apart.

### Example 5.6.2

From your text: Pg. 319 #6

The trunk of a leaning tree makes an angle of  $12^\circ$  with the vertical. To prevent the tree from falling over, a 35.0 m rope is attached to the top of the tree and is pegged into level ground some distance away. If the tree is 20.0 m from its base to its top, calculate the angle the rope makes with the ground to the nearest degree.

Note: the given angle ( $102^\circ$ ) is obtuse  $\Rightarrow$  No SSA case



$$\frac{\sin(\theta)}{20} = \frac{\sin(102)}{35}$$

$$\Rightarrow \sin(\theta) = \frac{(20)(\sin(102))}{35}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{(20)(\sin(102))}{35}\right)$$
$$= 34^\circ$$

$\therefore$  The rope makes an angle of  $34^\circ$  w/ the ground.

### Success Criteria

- I can recognize when the sine law applies and use it to solve for an unknown value
- I can identify, given S-S-A, that there may be two solutions (the ambiguous case)