Unit 5 – Trigonometric Ratios

5.8: 3D Problems

Learning Goal: We are learning to use trigonometry to solve 3-dimensional problems.

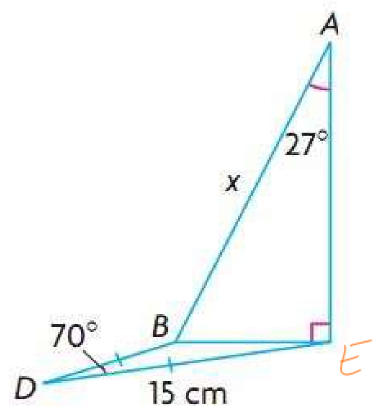
We will be using SOH CAH TOA, the Sine Law, and the Cosine Law for these problems. We'll jump right in by solving some problems since we already know how to use the various techniques! **One thing to keep in mind, though, is that these sorts of problems can be difficult to draw, or even simply visualize because we are working in 3D! Art specialists – rejoice!**

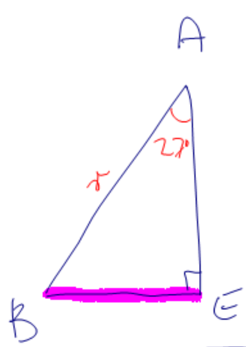
Key: look for commonality || Very helpful to separate the Δ's

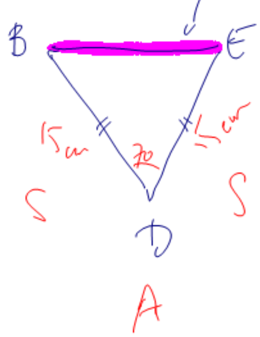
Example 5.8.1

From your text: Pg. 332 #4b

Solve for x







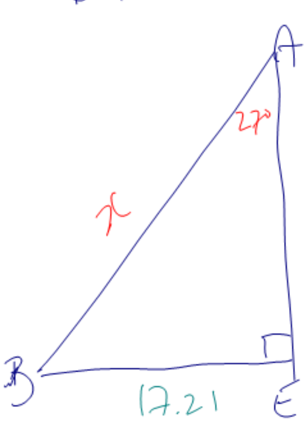
find BE – the common side

$$BE^2 = 15^2 + 15^2 - 2(15)(15)\cos(70)$$

$$\Rightarrow BE = \sqrt{\quad '' \quad}$$

$$\doteq 17.21$$

Back to ΔABE



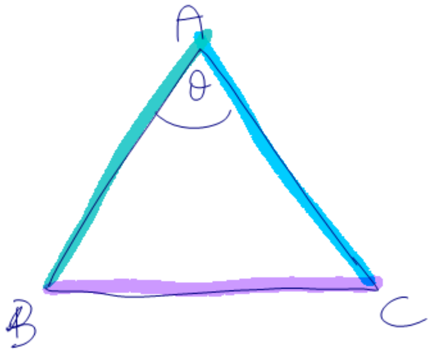
Using SOH CAH TOA

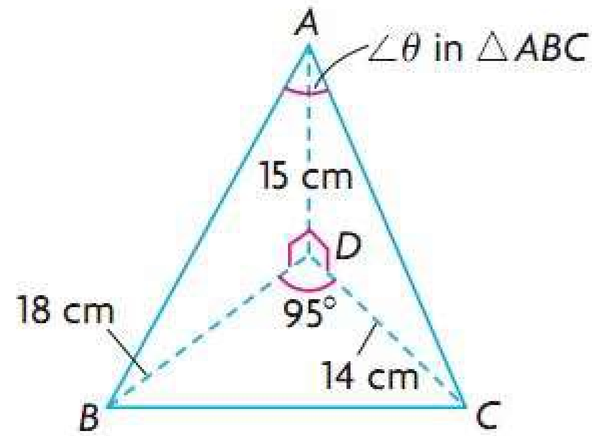
$$\sin(27) = \frac{17.21}{x}$$

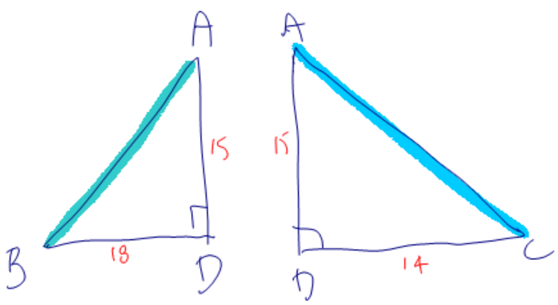
$$x \cdot \sin(27) = 17.21$$

$$x = \frac{17.21}{\sin(27)} \doteq 38 \text{ units.}$$

d) Solve for θ







In $\triangle ABD$ we use pyth. thm.

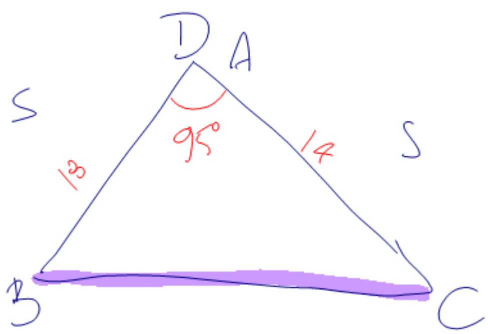
$$AB^2 = 18^2 + 15^2$$

$$AB = \sqrt{18^2 + 15^2} \doteq 23.4 \text{ cm}$$

In $\triangle ADC$

$$AC^2 = 15^2 + 14^2$$

$$AC = \sqrt{15^2 + 14^2} \doteq 20.5 \text{ cm}$$

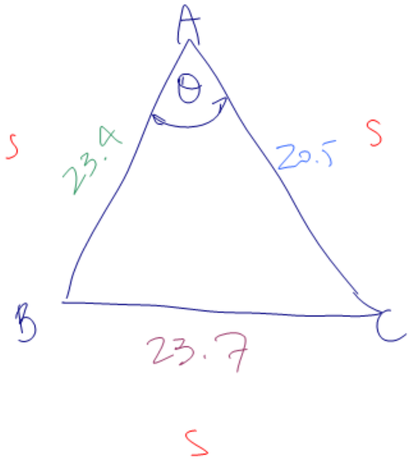


By Cosine law $BC^2 = 18^2 + 14^2 - 2(18)(14)\cos(95)$

$BC = \sqrt{\quad '' \quad}$

$\doteq 23.7$ cm.

∴ In $\triangle ABC$ we have



$$\cos(\theta) = \frac{(23.7^2 - 23.4^2 - 20.5^2)}{(-2(23.4)(20.5))}$$ Bar

$$\theta = \cos^{-1}\left(\frac{(23.7^2 - 23.4^2 - 20.5^2)}{(-2(23.4)(20.5))}\right) \doteq 65°$$

Example 5.8.2

From your text: Pg. 333 #5

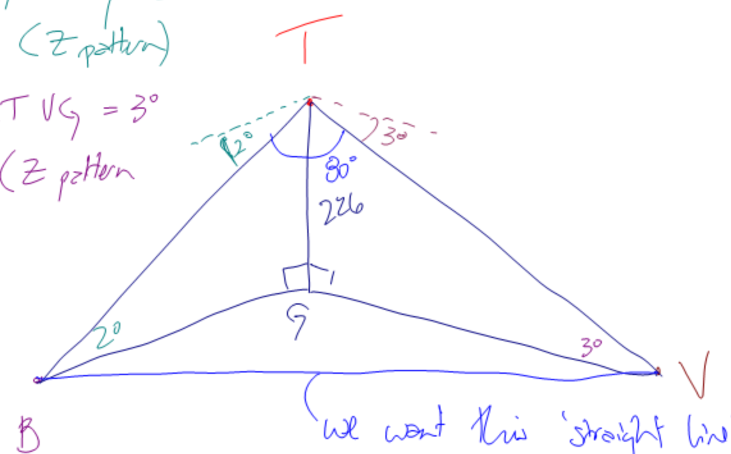
While Travis and Bob were flying a hot-air balloon from Beamsville to Vineland in southwestern Ontario, they decided to calculate the straight-line distance, to the nearest metre, between the two towns.

- From an altitude of 226 m, they simultaneously measured the angle of depression to Beamsville as 2° and to Vineland as 3°.
- They measured the angle between the lines of sight to the two towns as 80°.

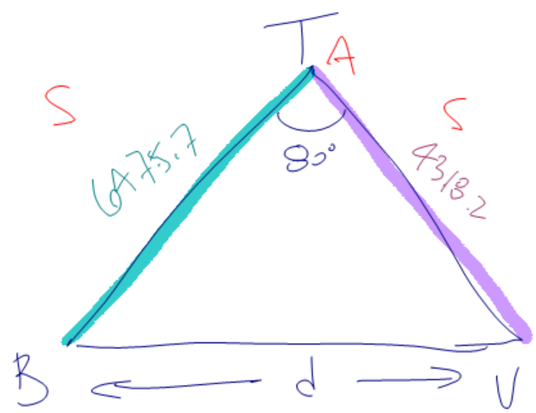
Is there enough information to calculate the distance between the two towns? Justify your reasoning with calculations.

$\angle TBG = 2°$ (Z pattern)

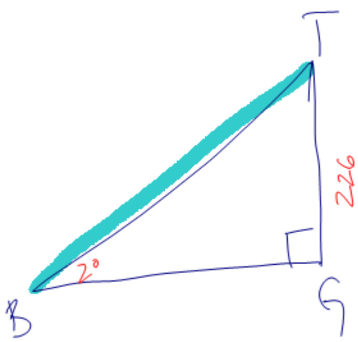
$\angle TVG = 3°$ (Z pattern)

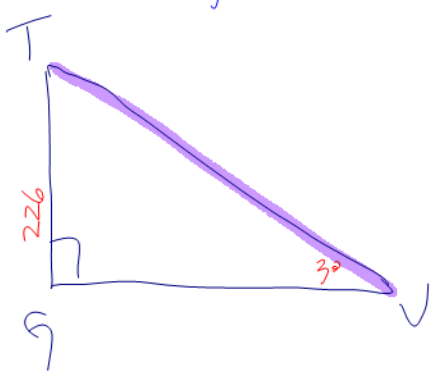


we want this 'straight line' distance



(cosine law)





$d^2 = 6475.7^2 + 4318.2^2 - 2(6475.7)(4318.2)\cos(80)$

$\Rightarrow d = \sqrt{\quad "\quad}$

$= 7125.8\ m$

In $\triangle TBG$ using SOH CAH TOA | Same in $\triangle TVG$.

$\sin(2) = \dfrac{226}{TB}$

$\Rightarrow TB = \dfrac{226}{\sin(2)}$

$= 6475.7\ m$

$\sin(3) = \dfrac{226}{TV}$

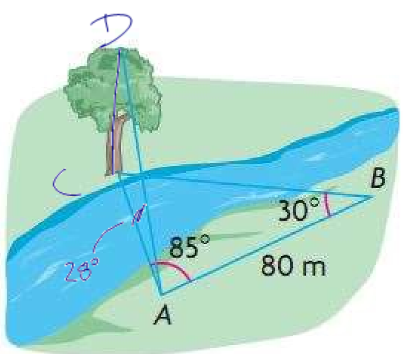
$\Rightarrow TV = \dfrac{226}{\sin(3)}$

$= 4318.2\ m$

$\therefore$ The distance between Beamsville & Vineland is about 7.13 km.

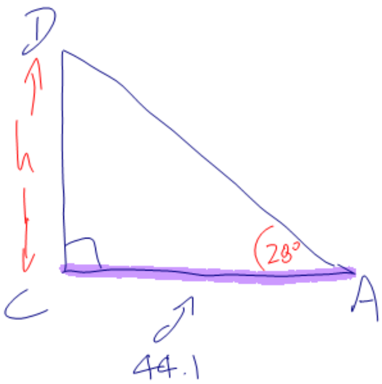
Example 5.8.3

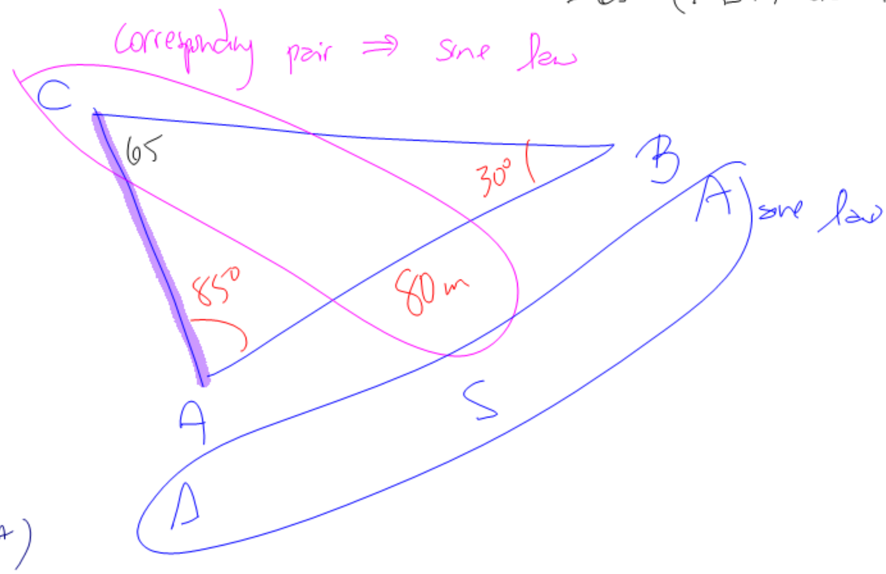
From your text: Pg. 334 #11



Bert wants to calculate the height of a tree on the opposite bank of a river. To do this, he lays out a baseline 80 m long and measures the angles as shown at the left. The angle of elevation from *A* to the top of the tree is 28°. Explain if this information helps Bert to calculate the height of the tree to the nearest metre. Justify your reasoning with calculations.

$\angle C = 180 - 85 - 30$
$= 65°$ (ASTT or +180)





In $\triangle ADC$ (SOH CAH TOA)

$$\tan(28) = \frac{h}{44.1}$$

$$\Rightarrow h = (44.1)(\tan(28))$$
$$= 23.4\text{ m}$$

$$\frac{CA}{\sin(30)} = \frac{80}{\sin(65)}$$

$$\Rightarrow CA = \frac{(80)(\sin(30))}{\sin(65)}$$
$$= 44.1\text{ m.}$$

∴ The tree is about 23.4 m.

Success Criteria:

- I can sketch, to the best of my ability, a representation of the question
- I can identify the correct method to solve the unknown(s) in a given problem