Solutions
(below the questions)

MCR3U – Unit 4: Trig Ratios

TRIG IDENTITIES PRACTICE

Prove the following Trigonometric Identities (on a separate piece of paper)

a) $\frac{\sin(\theta)}{\cos(\theta)\cdot\tan(\theta)} = 1$

b) $\sin(x)\cdot\tan(x) + \frac{1}{\cos(x)} = \frac{\sin^2(x)+1}{\cos(x)}$

c) $\frac{\sin^2(x)}{\tan^2(x)} = 1 - \sin^2(x)$

d) $\sin^2(\theta) = \frac{\tan^2(\theta)}{1+\tan^2(\theta)}$

e) $\sin(A) + \frac{\cos(A)}{\tan(A)} = \frac{1}{\cos(A)\cdot\tan(A)}$

f) $\tan(\theta) + \cot(\theta) = \csc(\theta)\cdot\sec(\theta)$

g) $(\cos(\phi) - \sin(\phi))^2 = 1 - 2\sin(\phi)\cdot\cos(\phi)$

h) $\sin^2(B) = \cos(B)\,[\sec(B) - \cos(B)]$

i) $\tan(x) + \frac{\cos(x)}{1+\sin(x)} = \frac{1}{\cos(x)}$

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a) $\frac{\sin(\theta)}{\cos(\theta)\cdot\tan(\theta)} = 1$

$LS: \frac{\sin(\theta)}{\cos(\theta)\cdot\tan(\theta)}$

$= \frac{\sin(\theta)}{\cancel{\cos(\theta)}\cdot\frac{\sin(\theta)}{\cancel{\cos(\theta)}}}$

$= \frac{\sin(\theta)}{\sin(\theta)} = 1 = RS$ □

b) $\sin(x)\cdot\tan(x) + \frac{1}{\cos(x)} = \frac{\sin^2(x)+1}{\cos(x)}$

$LS = \sin(x)\cdot\tan(x) + \frac{1}{\cos(x)}$

$= \frac{\sin(x)}{1}\cdot\frac{\sin(x)}{\cos(x)} + \frac{1}{\cos(x)}$

$= \frac{\sin^2(x)}{\cos(x)} + \frac{1}{\cos(x)}$ (common denom.)

$= \frac{\sin^2(x)+1}{\cos(x)} = RS$ □

c) $\dfrac{\sin^2(x)}{\tan^2(x)} = 1 - \sin^2(x)$

$$LS = \frac{\sin^2(x)}{\tan^2(x)}$$

$$= \frac{\frac{\sin^2(x)}{1}}{\frac{\sin^2(x)}{\cos^2(x)}}$$

flip and x

$$= \frac{\cancel{\sin^2(x)}}{1} \times \frac{\cos^2(x)}{\cancel{\sin^2(x)}}$$

$$= \cos^2(x)$$

pythagorean trig. identity

$$= 1 - \sin^2(x) = RS \ \square$$

d) $\sin^2(\theta) = \dfrac{\tan^2(\theta)}{1+\tan^2(\theta)}$

$$RS = \frac{\tan^2(\theta)}{1+\tan^2(\theta)}$$

$$= \frac{\frac{\sin^2(\theta)}{\cos^2(\theta)}}{1 + \frac{\sin^2(\theta)}{\cos^2(\theta)}}$$

get a common denominator in the denominator "$\cos^2(\theta)$" in this case

$\times \frac{\cos^2\theta}{\cos^2\theta}$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}} \quad \cos^2\theta + \sin^2\theta = 1!$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \frac{\cancel{\cos^2\theta}}{1} = \sin^2(\theta)$$

flip and x

$$= LS \ \square$$

e) $\sin(A) + \dfrac{\cos(A)}{\tan(A)} = \dfrac{1}{\cos(A)\cdot\tan(A)}$

$$LS = \sin(A) + \frac{\cos(A)}{\tan(A)}$$

$$= \sin(A) + \frac{\cos(A)}{\frac{\sin(A)}{\cos(A)}}$$

$$= \sin(A) + \frac{\cos(A)}{1} \times \frac{\cos(A)}{\sin(A)}$$

$$= \sin(A) + \frac{\cos^2(A)}{\sin(A)}$$

(common denom. "$\sin(A)$")

$$= \frac{\sin(A)\cdot\sin(A)}{\sin(A)} + \frac{\cos^2(A)}{\sin(A)}$$

$$= \frac{\sin^2(A) + \cos^2(A)}{\sin(A)} = \frac{1}{\sin(A)}$$

(moving to the other side)

$$RS = \frac{1}{\cos(A)\cdot\tan(A)}$$

$$= \frac{1}{\cancel{\cos(A)} \cdot \frac{\sin(A)}{\cancel{\cos(A)}}}$$

$$= \frac{1}{\sin(A)}$$

$$= LS \ \square$$

f) $\tan(A) + \cot(A) = \csc(A) \cdot \sec(A)$

$$LS = \tan(A) + \cot(A)$$

$$= \frac{\sin(A)}{\cos(A)} + \frac{\cos(A)}{\sin(A)}$$

(Common denom "$\cos(A) \cdot \sin(A)$")

$\times \frac{\sin A}{\sin A}$ $\qquad$ $\times \frac{\cos A}{\cos A}$

$$= \frac{\sin^2(A)}{\cos(A)\sin(A)} + \frac{\cos^2(A)}{\cos(A)\sin(A)}$$

$$= \frac{\sin^2(A) + \cos^2(A)}{\cos(A) \cdot \sin(A)}$$

$$= \frac{1}{\cos(A) \cdot \sin(A)}$$

$$RS = \csc(A) \cdot \sec(A)$$

$$= \frac{1}{\sin(A)} \cdot \frac{1}{\cos(A)}$$

$$= \frac{1}{\sin(A) \cdot \cos(A)}$$

$$= LS \ \square$$

g) $(\cos(\phi) - \sin(\phi))^2 = 1 - 2\sin(\phi) \cdot \cos(\phi)$

$$LS = (\cos(\phi) - \sin(\phi))^2$$

$$= (\cos(\phi) - \sin(\phi))(\cos(\phi) - \sin(\phi)) \quad \text{(Foil)}$$

$$= \cos^2(\phi) - \cos(\phi)\sin(\phi) - \sin(\phi)\cos(\phi) + \sin^2(\phi)$$

$$= \cos^2(\phi) - 2\cos(\phi) \cdot \sin(\phi) + \sin^2(\phi) \quad = 1$$

$$= 1 - 2\cos(\phi)\sin(\phi)$$

$$= RS \ \square$$

h) $\sin^2(B) = \cos(B)\,[\sec(B) - \cos(B)]$

$$RS = \cos(B)\left(\sec(B) - \cos(B)\right)$$

$$= \cos(B)\cdot\sec(B) - \cos^2(B)$$

$$= \cos(B)\cdot\frac{1}{\cos(B)} - \cos^2(B)$$

$$= 1 - \cos^2(B)$$

$$= \sin^2(B) = LS \square$$

i) $\tan(x) + \frac{\cos(x)}{1+\sin(x)} = \frac{1}{\cos(x)}$

$$LS = \tan(x) + \frac{\cos(x)}{1+\sin(x)}$$

$$= \frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{1+\sin(x)}$$ (common denom. "$(\cos(x))(1+\sin(x))$)

$$= \frac{\sin(x)}{\cos(x)}\cdot\frac{(1+\sin(x))}{(1+\sin(x))} + \frac{\cos(x)}{(1+\sin(x))}\cdot\frac{\cos(x)}{\cos(x)}$$

$$= \frac{\sin(x)(1+\sin(x)) + \cos^2(x)}{(\cos(x))(1+\sin(x))}$$

$$= \frac{\sin(x) + \underbrace{\sin^2(x) + \cos^2(x)}_{=1}}{(\cos(x))(1+\sin(x))}$$

$$= \frac{\cancel{\sin(x) + 1}}{(\cos(x))\cancel{(1+\sin(x))}}$$

$$= \frac{1}{\cos(x)}$$

$$= RS \square.$$