A∞Ω
MATH@TD

Unit 6 – Sinusoidal Functions

6.1 – Properties of Periodic Functions

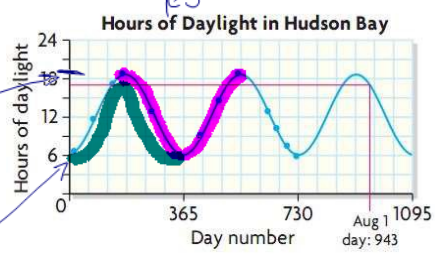
Learning Goal: We are learning to interpret and describe graphs that repeat at regular intervals.

Definition 6.1.1

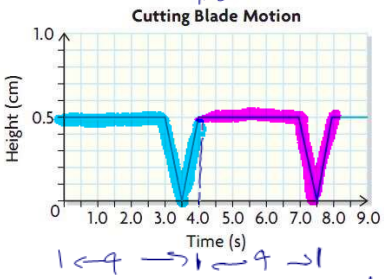
A **PERIODIC FUNCTION** is one in which *the functional values repeat*, in a predictable way.

e.g. Consider the following pictures: Determine which are periodic.

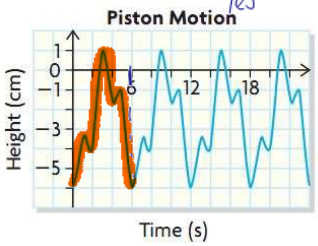
a) Yes



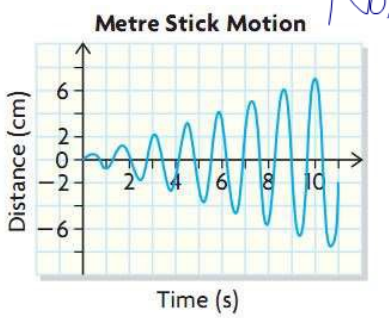
b) Yes



c) Yes



d) No.



Definition 6.1.2

The **Period** of a periodic function is the amount of the **domain values** where **one cycle** takes place.

↳ a repeated "shape" in a sketch

Example 6.1.1

Determine the periods of the above periodic functions:

a) 365 days b) 4 seconds c) 6 seconds

Definition 6.1.3

a) The **Amplitude** of a periodic function is half of the distance between a maximum value and a minimum value.

$$\text{Amplitude} = \frac{\max - \min}{2}$$

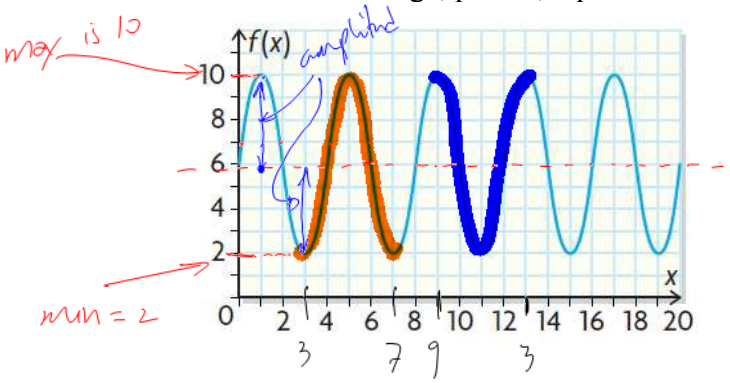
or amplitude is the distance from the central axis to a max or a min

b) The **Central Axis** is half way between the maximum value and the minimum value.

The equation of The Central Axis is given by $y = \frac{\max + \min}{2}$.

Example 6.1.1

Determine the range, period, equation of the axis, and amplitude of the function shown.



max is 10

min = 2

central

Range = $\{f(x) \in \mathbb{R} \mid 2 \le f(x) \le 10\}$ ($\min \le f(x) \le \max$)

the y-values that are a part of the f-

Period:

3 7 9 13

|←P→| |←4→|

distance is 4

$\Rightarrow$ period = 4 units

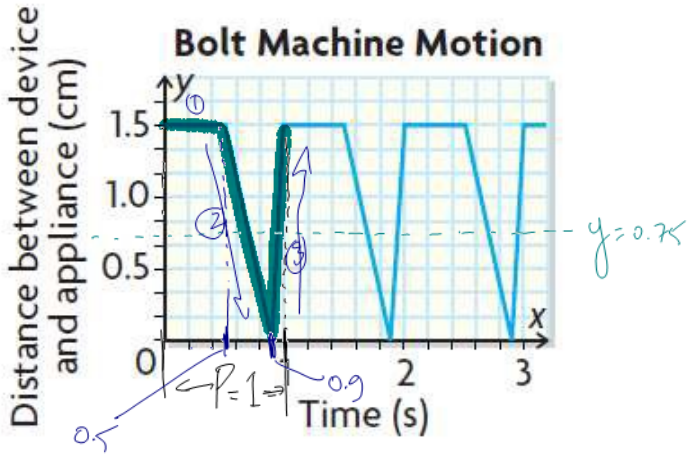
C.A. $y = \frac{\max + \min}{2} = \frac{10 + 2}{2}$

$y = 6$

amplitude $a = \frac{\max - \min}{2} = \frac{10 - 2}{2} = 4$

Example 6.1.2

3. The motion of an automated device for attaching bolts to a household appliance on an assembly line can be modelled by the graph shown at the left.
   a) What is the period of one complete cycle?
   b) What is the maximum distance between the device and the appliance?
   c) What is the range of this function?
   d) If the device can run for five complete cycles only before it must be turned off, determine the domain of the function.
   e) Determine the equation of the axis.
   f) Determine the amplitude.
   g) There are several parts to each complete cycle of the graph. Explain what each part could mean in the context of "attaching the bolt."



a) 1 second

b) 1.5 cm.

c) Range = $\{y \in \mathbb{R} \mid 0 \le y \le 1.5\}$

d) $D = \{x \in \mathbb{R} \mid 0 \le x \le 5\}$

e) C.A: $y = \frac{\max + \min}{2}$

$= \frac{1.5 + 0}{2}$

$y = 0.75$

f) $a = \frac{\max - \min}{2}$

$= \frac{1.5 - 0}{2}$

$= 0.75$

g) Part ① $0 \le x \le 0.5$ sec the machine is 'resting'

Part ② $0.5 \le x \le 0.9$ the machine attaches the bolt

Part ③ $0.9 \le x \le 1$ the machine returns to rest position.

Success Criteria:

- I can find the range, period, central axis, and amplitude of a periodic function
- I can determine IF a function is periodic