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MATH@TD

Sine and cosine

parent fn

$f(\theta) = \sin(\theta)$

$f(\theta) = \cos(\theta)$

Unit 6 – Sinusoidal Functions

6.5 – Sketching Sinusoidal Functions

Learning Goal: We are learning to sketch the graphs of sinusoidal functions using transformations.

By this point in your illustrious High School careers, you have a solid understanding of Transformations of Functions in general. In terms of the sinusoidal functions Sine and Cosine in particular, the concepts are as you expect, but the transformations have specific meanings relating to nature of the sinusoidal "wave".

General Form of the Sine and Cosine Functions

$$f(\theta) = a\sin(k(\theta - d)) + c \qquad g(\theta) = a\cos(k(\theta - d)) + c$$

	Transformation	Properties
vertical stretch	$a =$ amplitude (If $a < 0$ (negative) then we also have a vertical flip.)	$a = \dfrac{\max - \min}{2}$ ↳ If we a sketch and we can see "max" & "min" values
horizontal stretch	$k =$ Period factor (If $k < 0$ then we also have a horizontal flip)	Period = $\dfrac{360}{k}$ $\Longrightarrow$ $k = \dfrac{360}{P}$
horizontal shift	$d =$ Phase shift	Note: To determine d you MUST factor "k" away from "θ & d" eg $f(\theta) = -2\cos(3\theta - 240) + 1$ $d \neq 240°$! STANDARD FORM $f(\theta) = -2\cos(3(\theta - 80)) + 1$ $d = 80°$!
vertical shift	$c =$ central axis	$c = \dfrac{\max + \min}{2}$ ↳ If we have a sketch where we can "see" max & min value

$$f(\theta) = a \, {}^{\sin}_{\cos}\left(k(\theta - d)\right) + c$$

Example 6.5.1

Determine the amplitude, period, phase shift and the equation of the central axis for:

a) $f(\theta) = 2\sin\left(1\theta + 60^\circ\right) + 1$ $k = 1$

Amplitude $= 2$

period $= \frac{360}{k} = \frac{360}{1} = 360^\circ$

Central axis: $y = 1$

phase shift: 60° left

b) $g(\theta) = 3\cos\left(2\theta - 90^\circ\right)$ $k = 2$

not in standard form

std. form: $g(\theta) = 3\cos\left(2(\theta - 45)\right) + 0$

amplitude: $a = 3$

$P = \frac{360}{k} = \frac{360}{2} = 180^\circ$

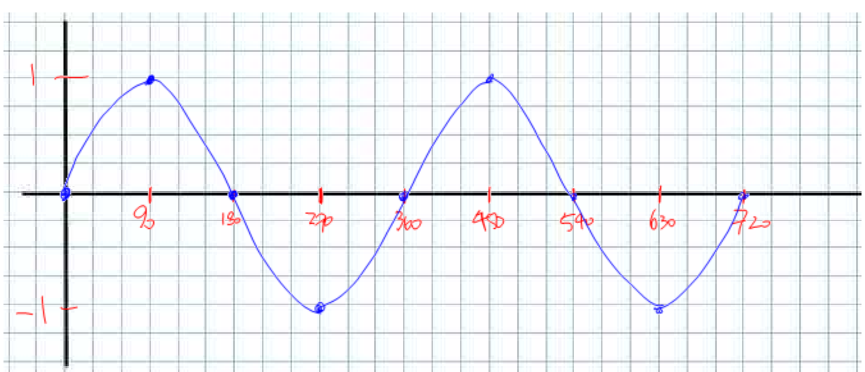
central axis: $y = 0$

phase shift: 45° right

The Graphs of Sin and Cos

Using the special angles of $\boxed{0^\circ, 90^\circ, 180^\circ, 270^\circ, \text{and } 360^\circ}$ axis angles, graph the following functions:

$f(\theta) = \sin(\theta) \quad 0^\circ \le \theta \le 720^\circ$



Patterns for sine/cosine.

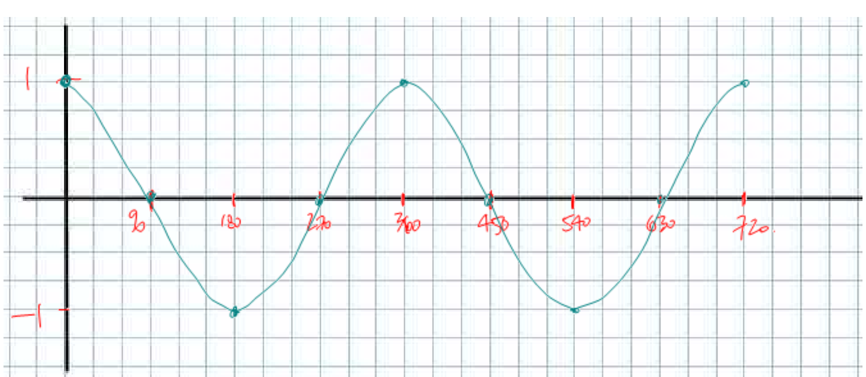
Sine:

"CA – max – CA – min – CA..."

If "a" is negative.

"CA – min – CA – max – CA..."

$g(\theta) = \cos(\theta) \quad 0^\circ \le \theta \le 720^\circ$



Cosine

"Max – CA – min – CA – max – ..."

If "a" is negative then the pattern is

"min – CA – max – CA – min – ..."

Example 6.5.2

Sketch $f(x)=\sin(x)$ and $g(x)=\sin(2x)$ for $0^\circ \le x \le 360^\circ$ on the same set of axes.

Analysis on $g(x)$. - 1 transformation

"$k=2$"

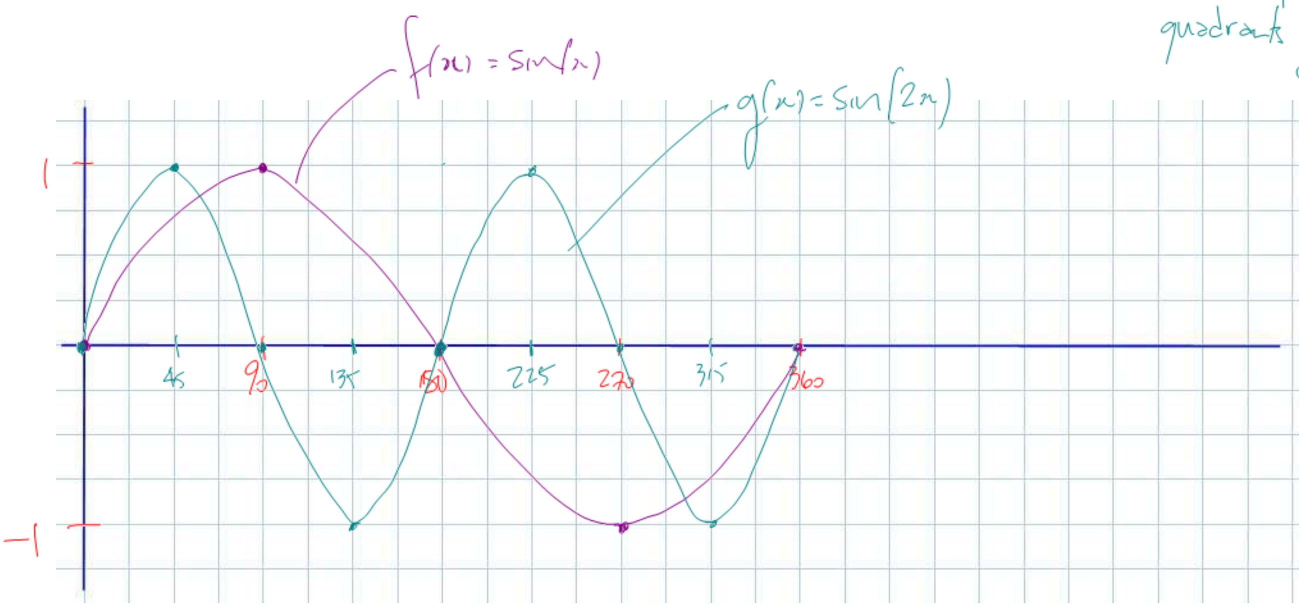
$P = \frac{360}{k} = \frac{360}{2} = 180^\circ$

"division of period into quadrants"

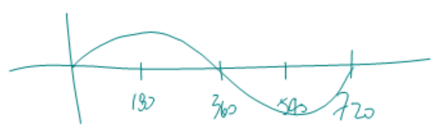
$d\theta = \frac{P}{4}$

$= \frac{180}{4}$

$= 45^\circ$



Now…what would $\sin\left(\frac{1}{2}x\right)$ look like?

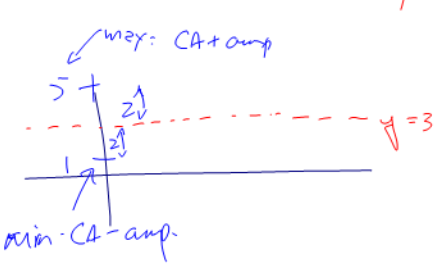


Notes about Domain and Range: Consider the function $f(x)=-2\cos(3x+90^\circ)+3$. Determine all the transformations for this function. Without graphing, determine the range of the function. Determine the domain of the function for: 1 cycle; 2 cycles; 3 cycles.

standard form $f(x) = -2\cos(3(x+30))+3$

amplitude	2 with a flip	period:	$\frac{360}{k} = \frac{360}{3} = 120^\circ$
central axis	$y=3$	phase shift	30° left.

Range: (picture)



$R: \{f(x) \in \mathbb{R} \mid 1 \le f(x) \le 5\}$

Domain:

1 cycle : $\{x \in \mathbb{R} \mid 0 \le x \le 120\}$

2 cycles $\{x \in \mathbb{R} \mid 0 \le x \le 240\}$

3 cycles $\{x \in \mathbb{R} \mid 0 \le x \le 360\}$

Example 6.5.3

Sketch $f(\theta) = -2\cos(\theta - 60^\circ) + 1$ on $0^\circ \le \theta \le 360^\circ$. State transformations, create tables, and state domain and range of the function.

start at min

k = 1 ∴ standard form already

We'll see 2 techniques

① "Old school" transformed Tables of Values

② Counting w/ a $d\theta$.

amp: 2 w/ a flip

P: $\frac{360}{1} = 360^\circ$

C.A. $y = 1$

phase shift 60° right

Parent: $g(\theta) = \cos(\theta)$

θ_P	g
0°	1
90°	0
180°	-1
270°	0
360°	1

Transformed

$\theta_T = \theta_P + 60$	$f = -2g + 1$
60	-1
150	1
240	3
330	1
420	-1

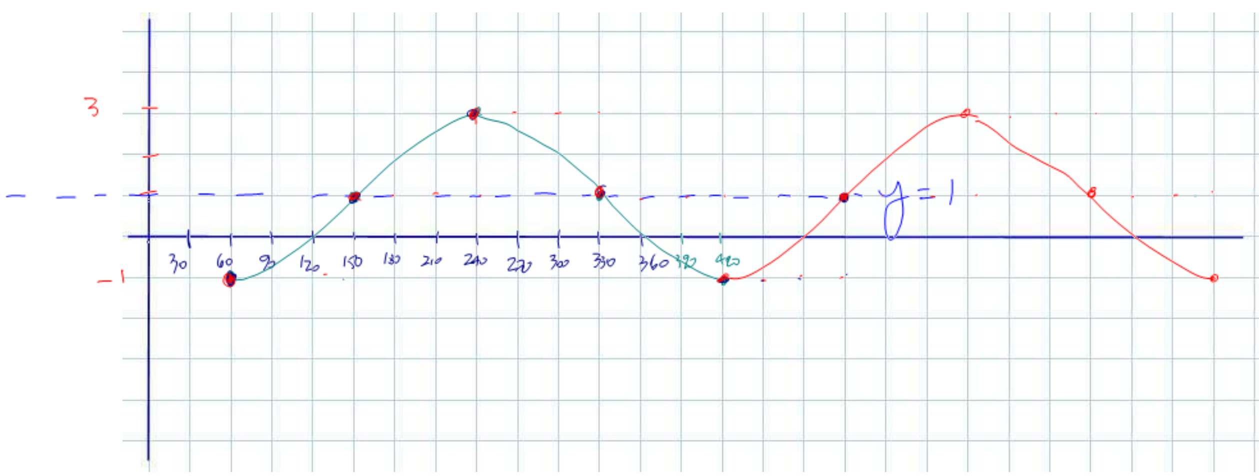
$d\theta = \frac{P}{4} = \frac{360}{4} = 90$

Phase Shift 60° right

scale: $\gcd\{d\theta, \text{P.S.}\} = \gcd\{90, 60\} = 30^\circ$

⇒ Phase Shift 2 boxes

⇒ $d\theta$ ⇒ 3 boxes



CA – max – CA – min – CA – max – . . .

factor k 'out'

Example 6.5.4

Sketch $f(\theta) = 3\sin(2\theta - 90) - 1$. State transformations, create tables, and state domain and range of the function.

k = 2

standard form $f(\theta) = 3\sin(2(\theta - 45)) - 1$

amplitude = 3

$P = \frac{360}{k} = \frac{360}{2} = 180°$

CA. : $y = -1$

Phase shift: 45 right

ToV.

parent: $g(\theta) = \sin\theta$

θ_p	g
0	0
90	1
180	0
270	−1
360	0

transformed $f(\theta)$

$\theta_T = \frac{1}{2}\theta_p + 45$	$f = 3g - 1$
45	−1
90	2
135	−1
180	−4
225	−1

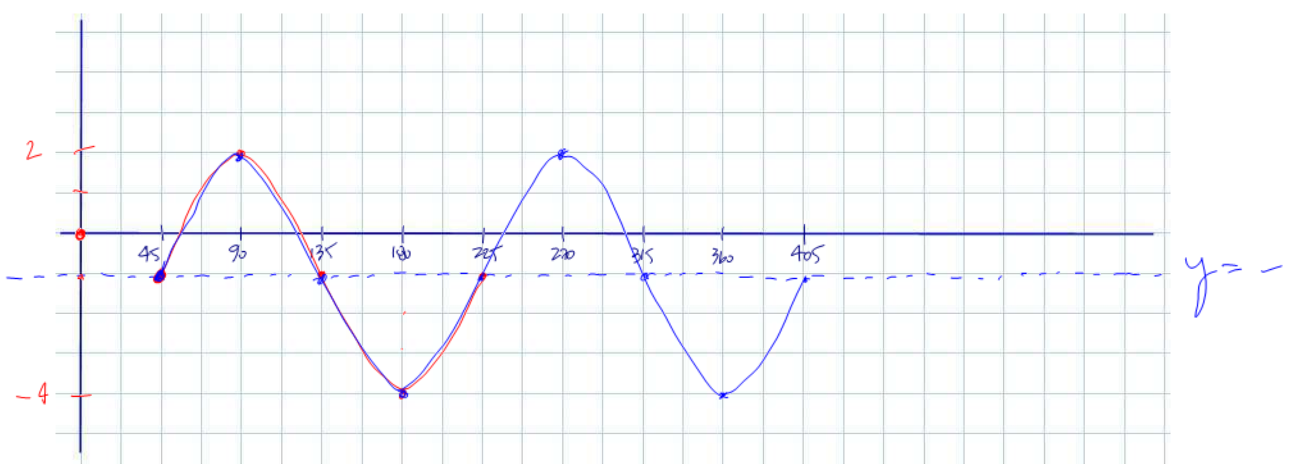
Counting technique

$P = 180$

$\Rightarrow \Delta\theta = \frac{P}{4} = \frac{180}{4} = 45°$

Phase shift 45° right

scale : $gcf\{\Delta\theta, P.S.\} = gcf\{45, 45\} = 45$



Success Criteria

- I can sketch the graph of a sinusoidal function by applying the transformations to the parent function.