Notes ① we have a cosine $\boxed{\text{if}}$ we "start" at a max or a min — neg. cosine
we have a sine $\boxed{\text{if}}$ we "start" at the central axis.

positive sine CA – max – CA
negative sine CA – min – CA

② "d" – the phase shift – is the horizontal distance our "start" point is away from the vertical axis || d = 0 if our "start" point is on the vertical axis.

Unit 6 – Sinusoidal Functions

6.6 – Models of Sinusoidal Functions

Learning Goal: We are learning to create a sinusoidal function from a graph or table of values.

In this section we will look at how to develop a sinusoidal function which can explain given information. In essence we will be writing sine or cosine functions based on given transformations.

Just as a reminder:

General Form of the Sine and Cosine Functions

$$f(\theta) = a\sin(k(\theta - d)) + c \qquad g(\theta) = a\cos(k(\theta - d)) + c$$

Another reminder (about the pattern of sinusoidal functions):

Sine functions "start" at the central axis and go up to a **max** if "*a*" is **positive**, or down to a **min** if "*a*" is **negative**.

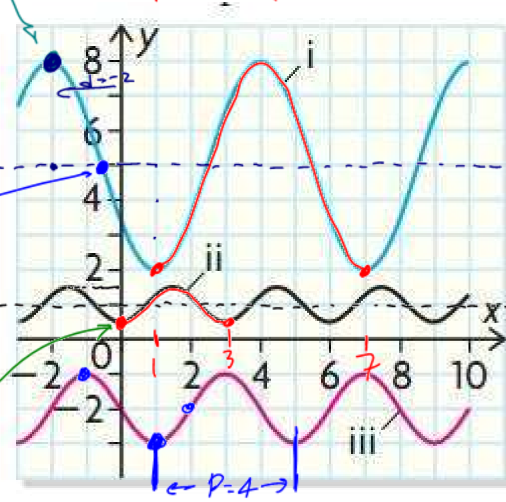
Cosine functions "start" at a **max** if "*a*" is **positive**, or at a **min** if "*a*" is **negative**.

$f(x) = a\cos(k(x-d)) + c$

Example 6.6.1

From your text: Pg. 391 #4a

Determine a sinusoidal equation for each function:



i) max = 8, min = 2

$a = \frac{max - min}{2} = \frac{8-2}{2} = 3$ | CA. $c = \frac{8+2}{2} = 5$

$k = \frac{360}{P} = \frac{360}{6} = 60$ || d = -2 for a positive cosine

the fn is $f(x) = +3\cos(60(x+2)) + 5$

or $f(x) = -3\cos(60(x-1)) + 5$

ii) max = 1.5, min = 0.5

$a = \frac{max - min}{2} = \frac{1}{2}$ | $c = \frac{max + min}{2} = 1 \Rightarrow y = 1$

start here at a min on the y-axis ⇒ negative cosine with d = 0

P = 3

$\Rightarrow k = \frac{360}{P} = \frac{360}{3} = 120$

$\boxed{\therefore g(x) = -\frac{1}{2}\cos(120x) + 1}$

iii) a = 1 c = -2

$k = \frac{360}{4} = 90$

d = 0 – neg sine
d = -2 + sine
d = +2 +ve sine
d = -1 + cosine
d = +1 – ve cosine

$\boxed{h(x) = -\sin(90x) - 2}$

$$f(x) = \underline{a}\ {}^{\sin}_{\cos}\left(\underline{k}\left(x - \underline{d}\right)\right) + \underline{c}$$

Example 6.6.2

From your text: Pg. 392 #5a)

5. For each table of data, determine the equation of the function that is the simplest model.

a)

x	0°	30°	60°	90°	120°	150°	180°
y	3	2	1	2	3	2	1

max → max

$P = 120.$

$k = \frac{360}{120}$

$= 3$

max = 3

min = 1

$a = \frac{3-1}{2} = 1$

$CA = \frac{3+1}{2} = 2$

d = 0 for a positive cosine.

k = 3

not needed

$$f(x) = \cos\left(3\left(x - 0°\right)\right) + 2$$

Example 6.6.3

From your text: Pg. 392 #6b)

6. Determine the equation of the cosine function whose graph has each of the following features.

	Amplitude (a)	Period ($k = \frac{360}{P}$)	Equation of the Axis (c)	Horizontal Translation (d)
a)	3	360°	$y = 11$	0°
b)	4	180°	$y = 15$	30° right

a) $k = \frac{360}{P} = \frac{360}{360} = 1$ $\quad f(x) = 3\cos(x) + 11$

b) $k = \frac{360}{P} = \frac{360}{180} = 2$ $\quad g(x) = 4\cos\left(2\left(x - 30\right)\right) + 15$

$$f(x) = \underline{a}\ {\sin \atop \cos}\left(\underline{k}\left(x-\underline{d}\right)\right)+\underline{c}$$

Example 6.6.4
A sinusoidal function has an amplitude of 4 units, a period of 120º, and a maximum at (0,9). Determine the equation of the function.

max at (0, 9)
↳ positive cosine.

Know

$a = 4$

$p = 120 \Rightarrow k = \frac{360}{p} = \frac{360}{120} = 3$

$d = 0$

$c = \text{max} - \text{amplitude}$
$= 9 - 4 = 5$ $\quad y = 5$

$$\boxed{\therefore f(x) = 4\cos(3x) + 5}$$

Success Criteria:
- I can determine the equation of a sinusoidal function based on information from a graph or table
- I can recognize when it is best to use a sine or cosine function (starting at a max (COSINE), or the central axis (SINE))