

$$h(t) = a \cos(k(t-d)) + c$$

↖ which sinusoidal f.

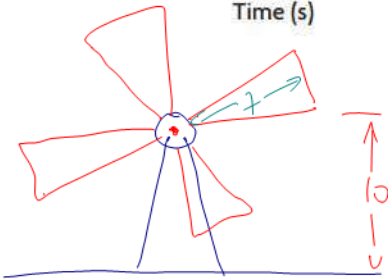
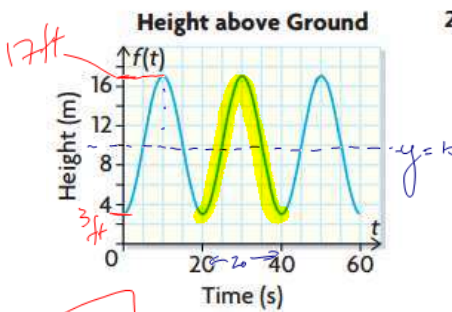
Unit 6 – Sinusoidal Functions

6.7 – Problem Solving with Sinusoidal Functions

Learning Goal: We are learning how to solve problems related to real-world applications of sinusoidal functions.

We can use the sinusoidal properties of **Period**, **Central Axis**, **Amplitude** and **Phase Shift** to describe and solve “real world” problems.

Example 6.7.1 (From the text: Pg. 398 #2)



2. Don Quixote, a fictional character in a Spanish novel, attacked windmills because he thought they were giants. At one point, he got snagged by one of the blades and was hoisted into the air. The graph shows his height above ground in terms of time.

- What is the equation of the ^{central} axis of the function, and what does it represent in this situation?
- What is the amplitude of the function, and what does it represent in this situation?
- What is the period of the function, and what does it represent in this situation?
- If Don Quixote remains snagged for seven complete cycles, determine the domain and range of the function.
- Determine the equation of the sinusoidal function.
- If the wind speed decreased, how would that affect the graph of the sinusoidal function?

↳ the time for a rotation increases ⇒ the period gets longer

a) C.A. $y = \frac{\text{max} + \text{min}}{2}$
 $= \frac{17 + 3}{2} = 10$
 $y = 10 \Rightarrow$ height of the axis of the windmill

b) $a = \frac{\text{max} - \text{min}}{2}$
 $= \frac{17 - 3}{2} = 7 \text{ ft}$

The windmill blades are 7 ft long.

c) $a = 7, c = 10, k = ?, d = ?$

c) $P = 20$ sec which is how long it takes for DQ. to make one rotation.

d) Range: $\{h(t) \in \mathbb{R} \mid \min = h(t) \leq \max\}$
 $= \{h(t) \in \mathbb{R} \mid 3 \leq h(t) \leq 17\}$

Domain (for 7 cycles)

$D: \{t \in \mathbb{R} \mid 0 \leq t \leq 140\}$ (time for 7 cycles) $(7)(20)$

$$k = \frac{360}{p} = \frac{360}{20} = 18, \quad d = 0 \quad (\text{DQ is "hooked at the min" at } t=0) \quad \text{neg. cosine}$$

$$\therefore \text{eqn: } \boxed{h(t) = -7 \cos(18t) + 10}$$

Example 6.7.2

A group of students is tracking a friend, John, who is riding a Ferris wheel. They know that John reaches a maximum height of 11m at 10s and then reaches a minimum height of 1m at 55s. How high is John after 2 minutes?

we want to calculate a number "h(2 min)"

Need a, k, d, c

we need a sinusoidal fcn "round and round"

$$a = \frac{\text{max} - \text{min}}{2} = \frac{11 - 1}{2} = 5$$

$$c = \frac{\text{max} + \text{min}}{2} = \frac{11 + 1}{2} = 6$$

Our eqn

$$\boxed{h(t) = 5 \cos(4(t - 10)) + 6}$$

n seconds

$$k = \frac{360}{p} = \frac{360}{90 \text{ sec.}} = 4$$

Note: we know John takes 55 - 10 = 45 seconds to move from a max to a min.

2 minutes = 120 seconds

$$h(120) = 5 \cos(4(120 - 10)) + 6 = 6.87 \text{ m.}$$

↳ 1/2 period

$$\Rightarrow \frac{1}{2} p = 45$$

$$\Rightarrow p = (45)(2) = 90 \text{ sec}$$

If we "start" at the max of 11 m at 10 sec

⇒ positive cosine with a phase shift of 10.

$$\boxed{d = 10}$$

Example 6.7.3 (Text pg. 396)

The top of a flagpole sways back and forth in high winds. The top sways 10 cm to the right (+10 cm) and 10 cm to the left (-10 cm) of its resting position and moves back and forth 240 times every minute. At $t = 0$, the pole was momentarily at its resting position. Then it started moving to the right.

- a) Determine the equation of a sinusoidal function that describes the distance the top of the pole is from its resting position in terms of time.

Need a, k, d, c

$$a = 10$$

$$d = 0$$

$$c = 0$$

$$k = \frac{360}{P}$$

$$= \frac{360}{\frac{1}{2}}$$

$$= 1440$$

$$P = \frac{1}{240} \text{ min/cycle}$$

240 cycles / min

$$= \frac{60}{240} = \frac{1}{4} = 0.25 \text{ seconds/cycle}$$

\therefore eqn $f(t) = 10 \sin(1440t)$

- b) How does the situation affect the domain and range?

Domain - the length of time the flagpole vibrates

Range - the wind speed will make the pole sway more/less depending on how fast the wind is.

- c) If the wind speed decreases slightly such that the sway of the top of the pole is reduced by 20%, what is the new equation of the sinusoidal function? Assume that the period remains the same.

"a" is reduced by 20% \Rightarrow we have 80% of the amplitude

$$\Rightarrow 0.8 \times 10 = 8 \text{ cm}$$

\therefore New eqn would $f(t) = 8 \sin(1440t)$

Success Criteria:

- I can create a sinusoidal function that represents information from a real-life scenario
- I can use the function to solve further problems