Unit 7 – Sequences and Series (Discrete Functions)

7.1 – Arithmetic Sequences

Learning Goal: We are learning to recognize the characteristics of arithmetic sequences, and express the general terms in a variety of ways.

Definition 7.1.1

A **mathematical sequence** is a list of numbers, usually with some kind of order. Each number is called a "term" of the sequence.

"way" of moving from one term to the next

e.g. $2 \xrightarrow{+5} 7 \xrightarrow{+5} 12 \xrightarrow{+5} 17 \xrightarrow{+5} 22 \xrightarrow{+5} 27, \ldots$

"term 1" → t_1, t_2, t_3, t_4, t_5, t_6

ellipsis "continue with the same pattern"

Definition 7.1.2

An **Arithmetic Sequence** is a sequence where each term differs from the previous term by a common difference *d*.

$n = 4$, $n-1 = 4-1 = 3$

$d = t_4 - t_3$ or $d = t_5 - t_4$ or $d = t_2 - t_1$

e.g. $1 \xrightarrow{+3} 4 \xrightarrow{+3} 7 \xrightarrow{+3} 10 \xrightarrow{+3} 13$

$d = 3$ (the difference between successive terms)

In general

$$\boxed{d = t_n - t_{n-1}}$$

where "n" is → index telling us which term we are considering

Definition 7.1.3

The general **term** of an arithmetic sequence, usually labelled t_n, is given by a formula. The subscript *n* gives the position of the term in the sequence, with one exception: the first term of a sequence, t_1, is called a.

t_n

$t_1, t_2, t_3, t_4, t_5, t_6, \ldots$

e.g. In the sequence $4, -3, -10, -17, -24, -31, \ldots$

$t_1 = a = 4$

$t_2 = -3$

$t_5 = -24$

$d = t_3 - t_2$

$= -10 - (-3)$

$= -7$

Note: members of a sequence are also called terms, or elements.

"t_n"

The $\boxed{\text{General Term}}$ of an Arithmetic Sequence

The formula can be arrived at using some simple logic. Consider some arithmetic sequence with first term a and common difference d. Then the sequence can be written:

t_1	t_2	t_3	t_4	t_5	...	t_{100}
a	$a+d$	t_2+d	t_3+d	$a+4d$		$a+99d$
		$=a+d+d$	$a+2d+d$			
		$=a+2d$	$a+3d$			

$t_n = a + (n-1)d$

$$\boxed{t_n = a+(n-1)d}$$

$\Downarrow$

This allows us to find any member of the sequence without writing out "a million" terms

Example 7.1.1

From your text: Pg. 424 # 1

Determine which sequences are arithmetic. For those that are, state the common difference.

a) 1, 5, 9, 13, 17, ... Yes $d=4$ ($+4$, $+4$, $+4$, $+4$)

b) 3, 7, 13, 17, 23, 27, ... ($+4$, $+6$, $+4$, $+6$, $+4$) No - not arithmetic.

c) 3, 6, 12, 24, ... ($\times 2$, $\times 2$, $\times 2$) No - not arithmetic

d) 59, 48, 37, 26, 15, ... (-11, -11, -11, -11) Yes $d=-11$

Example 7.1.2

From your text: Pg 424 #6

Determine the recursive formula and the general term for the arithmetic sequence in which

a) the first term is 19 and consecutive terms increase by 8

b) $t_1 = 4$ and consecutive terms decrease by 5

c) the first term is 21 and the second term is 26

d) $t_4 = 35$ and consecutive terms decrease by 12

Note: $t_4 - t_1 = (a+3d) - (a)$

$t_4 - t_1 = 3d$ ($4-1$)

A recursive formula requires 2 things

① A place to start

$t_1 = a$

② A way to move from one term to the next.

a) $t_1 = 19$, $d = 8$

recursive formula

① $\boxed{t_1 = 19}$

② $t_n = t_{n-1} + d$

$\boxed{t_n = t_{n-1} + 8}$

general term

$t_n = a + (n-1)d$

$\Rightarrow t_n = 19 + (n-1)8$

b) $t_1 = 4$, $d = -5$

recursive

$\boxed{\begin{array}{l} t_1 = 4 \\ t_n = t_{n-1} - 5 \end{array}}$

general

$t_n = a + (n-1)d$

$t_n = 4 + (n-1)(-5)$

$\Rightarrow \boxed{t_n = -5n + 9}$

d) $t_4 = 35$, $d = -12$

$t_4 - t_1 = 3d$

$35 - t_1 = 3(-12)$

$35 - t_1 = -36$

$t_1 = 71$

recursive

$\boxed{\begin{array}{l} t_1 = 71 \\ t_n = t_{n-1} - 12 \end{array}}$

general

$t_n = a + (n-1)d$

$\Rightarrow t_n = 71 + (n-1)(-12)$

$= -12n + 83$

arithmetic sequences "look like" lines

$\Rightarrow t_n = 19 + 8n - 8$

$\boxed{t_n = 8n + 11}$ simplified form of general term

"d" is the "slope of the line"

Example 7.1.3

From your text: Pg. 425 #9a

i) Determine whether each general term defines an arithmetic sequence.
ii) If the sequence is arithmetic, state the first five terms and the common difference.

a) $t_n = 8 - 2n$

$t_n = \boxed{-2}n + 8$

This is arithmetic with $d = -2$

$t_1 = -2(1) + 8$
$= 6$

ii) t_1, t_2, t_3, t_4, t_5

-2 -2 -2 -2

$6, 4, 2, 0, -2$

Example 7.1.4

From your text: Pg 424 #13b

Determine the number of terms in the arithmetic sequence

$-20, -25, -30, \ldots, -205$

t_1 t_2 t_3 $\cdots$

-5 -5

t_n, where "n" is the number of terms

$a = -20$

$d = t_2 - t_1$
$= -25 - (-20)$
$= -5$

$t_n = a + (n-1)d$

$\Rightarrow -205 = (-20) + (n-1)(-5)$

$\Rightarrow -205 = -20 - 5n + 5$

$5n = 190$

$n = \frac{190}{5}$
$= 38$

$\therefore$ There are 38 terms in the sequence

Example 7.1.5

Given an arithmetic sequence with $t_7 = 25$ and $t_{20} = 77$ determine the general term for the sequence, and also determine t_{150}.

↳ need (a) & (d)

$t_{20} - t_7$
$= (a + 19d) - (a + 6d)$
$= 13d$

$t_{20} - t_7 = 13d$
$\Rightarrow 77 - 25 = 13d$
$52 = 13d$

$d = \frac{52}{13}$
$= 4$

$t_7 = a + 6d$
$\Rightarrow 25 = a + 6(4) \Rightarrow a = 1$

Success Criteria:

- I can identify when a sequence is arithmetic, by seeing if it has a common difference
- I can use the General Term Formula to develop an equation for an arithmetic sequence
- I can use the General Term to find any term in a sequence OR to find out how many terms are in a sequence
- I can recognize that an arithmetic sequence is always a linear function

$\therefore t_n = a + (n-1)d$
$\Rightarrow t_n = 1 + (n-1)(4)$

$t_{150} = 1 + (149)(4)$
$= 597$

$\Rightarrow$ $\boxed{t_n = 4n - 3}$ simplified form