

# Unit 7 – Sequences and Series (Discrete Functions)

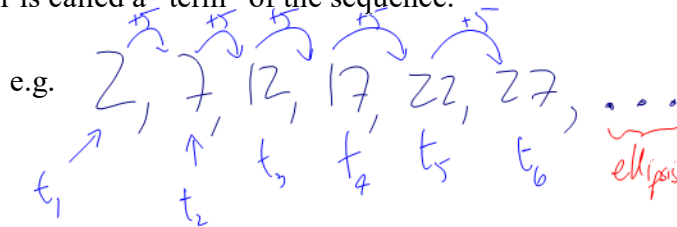
## 7.1 – Arithmetic Sequences

**Learning Goal:** We are learning to recognize the characteristics of arithmetic sequences, and express the general terms in a variety of ways.

### Definition 7.1.1

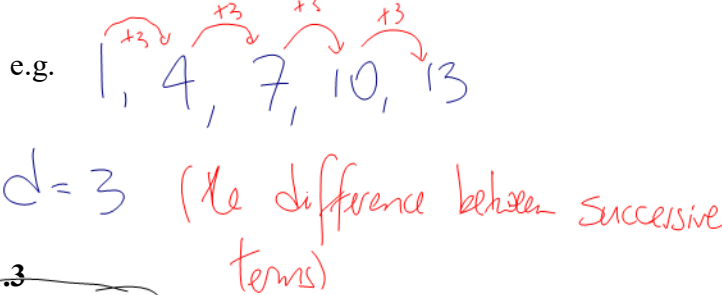
A **mathematical sequence** is a list of numbers, usually with some kind of **order**. Each number is called a “term” of the sequence.

“way” of moving from one term to the next



### Definition 7.1.2

An **Arithmetic Sequence** is a sequence where each term differs from the previous term by a common difference  $d$ .



In general

$$d = t_n - t_{n-1}$$

where “n” is index telling us which term we are considering

### Definition 7.1.3

The general **term** of an arithmetic sequence, usually labelled  $t_n$ , is given by a formula. The subscript  $n$  gives the position of the term in the sequence, with one exception: the first term of a sequence,  $t_1$ , is called  $a$ .

e.g. In the sequence  $t_1, t_2, t_3, t_4, t_5, t_6, \dots$   
 4, -3, -10, -17, -24, -31, ...

$$\begin{aligned}
 t_1 &= a = 4 \\
 t_2 &= -3 \\
 t_5 &= -24 \\
 d &= t_3 - t_2 \\
 &= -10 - (-3) \\
 &= -7
 \end{aligned}$$

Note: members of a sequence are also called terms, or elements

$n=4$   
 $n-1=4-1=3$

$$d = t_4 - t_3$$

or

$$d = t_5 - t_4$$

or

$$d = t_2 - t_1$$

$t_n$

# The General Term of an Arithmetic Sequence

The formula can be arrived at using some simple logic. Consider some arithmetic sequence with first term  $a$  and common difference  $d$ . Then the sequence can be written:

$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	...	$t_{100}$
$a$	$a+d$	$t_2+d$ $= a+d+d$ $= a+2d$	$t_3+d$ $= a+2d+d$ $= a+3d$	$a+4d$		$a+99d$

$$t_n = a + (n-1)d$$

$$t_n = a + (n-1)d$$

### Example 7.1.1

From your text: Pg. 424 # 1

Determine which sequences are arithmetic. For those that are, state the common difference.

- a) 1, 5, 9, 13, 17, ... Yes  $d=4$
- b) 3, 7, 13, 17, 23, 27, ... No - not arithmetic
- c) 3, 6, 12, 24, ... No - not arithmetic
- d) 59, 48, 37, 26, 15, ... Yes  $d=-11$

↓

This allows us to find any member of the sequence without writing out "a million" terms

### Example 7.1.2

From your text: Pg 424 #6

Determine the recursive formula and the general term for the arithmetic sequence in which

- a) the first term is 19 and consecutive terms increase by 8
- b)  $t_1 = 4$  and consecutive terms decrease by 5
- c) the first term is 21 and the second term is 26
- d)  $t_4 = 35$  and consecutive terms decrease by 12

Note:  $t_4 - t_1 = (a+3d) - (a)$   
 $t_4 - t_1 = 3d = 35 - 21$

A recursive formula requires 2 things

① A place to start  $t_1 = a$

② A way to move from one term to the next

a)  $t_1 = 19, d = 8$

recursive formula

①  $t_1 = 19$

②  $t_n = t_{n-1} + 8$

general term

$t_n = a + (n-1)d$

$t_n = 19 + (n-1)8$

b)  $t_1 = 4, d = -5$

recursive

$t_1 = 4$   
 $t_n = t_{n-1} - 5$

general

$t_n = a + (n-1)d$

$t_n = 4 + (n-1)(-5)$

$t_n = -5n + 9$

d)  $t_4 = 35, d = -12$

$t_4 - t_1 = 3d$

$35 - t_1 = 3(-12)$

$35 - t_1 = -36$

$t_1 = 71$

recursive

$t_1 = 71$   
 $t_n = t_{n-1} - 12$

general

$t_n = a + (n-1)d$

$t_n = 71 + (n-1)(-12)$

$= -12n + 83$

arithmetic sequences  
"look like" lines

$$\Rightarrow t_n = 19 + 8n - 8$$

$$t_n = 8n + 11$$

simplified form of general term  
"d" is the "slope of the line"

**Example 7.1.3**

From your text: Pg. 425 #9a

- i) Determine whether each general term defines an arithmetic sequence.
- ii) If the sequence is arithmetic, state the first five terms and the common difference.

$$t_1 = -2(1) + 8 = 6$$

a)  $t_n = 8 - 2n$

$$t_n = -2n + 8$$

This is arithmetic with  $d = -2$

ii)  $t_1, t_2, t_3, t_4, t_5$

6, 4, 2, 0, -2

(Arrows showing a common difference of -2 between terms)

**Example 7.1.4**

From your text: Pg 424 #13b

Determine the number of terms in the arithmetic sequence

$a = -20$

$-20, -25, -30, \dots, -205$

$d = t_2 - t_1$

$= -25 - (-20)$   
 $= -5$

$t_n$ , where "n" is the number of terms

$$t_n = a + (n-1)d$$

$$\Rightarrow -205 = (-20) + (n-1)(-5)$$

$$\Rightarrow -205 = -20 - 5n + 5$$

$$5n = 190$$

$$n = \frac{190}{5} = 38$$

$\therefore$  There are 38 terms in the sequence

**Example 7.1.5**

Given an arithmetic sequence with  $t_7 = 25$  and  $t_{20} = 77$  determine the general term for the sequence, and also determine  $t_{150}$ .

$$t_{20} - t_7 = 13d$$

$$= (a + 19d) - (a + 6d)$$

$$= 13d$$

$$\Rightarrow 77 - 25 = 13d$$

$$52 = 13d$$

$$d = \frac{52}{13} = 4$$

$$t_7 = a + 6d$$

$$\Rightarrow 25 = a + 6(4) \Rightarrow a = 1$$

**Success Criteria:**

- I can identify when a sequence is arithmetic, by seeing if it has a common difference
- I can use the General Term Formula to develop an equation for an arithmetic sequence
- I can use the General Term to find any term in a sequence OR to find out how many terms are in a sequence
- I can recognize that an arithmetic sequence is always a linear function

$$\therefore t_n = a + (n-1)d$$

$$\Rightarrow t_n = 1 + (n-1)(4)$$

$$t_{150} = 1 + (149)(4)$$

$$= 597$$

→  $t_n = 4n - 3$  simplified form