Unit 7 – Sequences and Series (Discrete Functions)

7.2 – Geometric Sequences

Learning Goal: We are learning the characteristics of geometric sequences and how to express the general terms in a variety of ways.

recursive

$t_1 = a$

$t_n = (t_{n-1}) + d$

$t_n = a + (n-1)d$ (general)

In the last lesson we considered sequences of the form:

$$3 \xrightarrow{+4} 7 \xrightarrow{+4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19 \xrightarrow{+4} 23, \ldots$$

This sequence is arithmetic because there is a common **difference** between successive terms. We can write the general term of the above sequence because we know the first term $(a = 3)$, and the common difference $(d = 4)$.

$$t_n = 3 + (n-1)(4) = 4n - 1$$

Note that if we "simplify" the general term, we can actually consider that simplification as a **function** of n!!

i.e. $f(n) = 4n - 1, \; n \in \mathbb{N}$

Consider the following sequence:

$$3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12 \xrightarrow{\times 2} 24 \xrightarrow{\times 2} 48, \ldots$$

This sequence is not arithmetic, but there is a discernible pattern as we look at moving from one term to the **next** term. In this case, we see that each new term is generated by multiplying the previous term by 2.

Such a sequence is called a **Geometric Sequence**. There isn't a common difference between two successive terms, but there is a **common ratio** (r) between two successive terms.

Comparing Two Successive Terms

Arithmetic	**Geometric**
$t_n - t_{n-1} = d$	$\dfrac{t_n}{t_{n-1}} = r$

General Term (Geometric)

$t_n = ar^{n-1}$

recursive formula

$\boxed{t_1 = a \quad t_n = (t_{n-1})r}$

The General Term of a Geometric Sequence

Again, using simple logic will allow us to arrive at a formula (or even a function depending on how you interpret things). Consider some geometric sequence with first term a and common ratio r. The sequence can be written:

$$t_1, \quad t_2, \quad t_3, \quad t_4, \quad \ldots, \quad t_n$$

$$(t_1)(r) \quad (t_2)(r) \quad (t_3)(r)$$

$$(ar)(r) \quad (ar^2)(r)$$

$$a, \quad ar, \quad ar^2, \quad ar^3, \quad \ldots, \quad \boxed{t_n = ar^{n-1}}$$

Example 7.2.1

From your text: Pg. 430 #1

Determine which sequences are geometric. For those that are, state the common ratio.

is arithmetic

a) 15, 26, 37, 48, ... No, not geometric

b) 5, 15, 45, 135, ... Yes $r = 3$

c) 3, 9, 81, 6561, ... No.

d) 6000, 3000, 1500, 750, 375, ... Yes $r = \frac{1}{2}$

Example 7.2.2

Determine the general term (t_n) and t_{10} of the geometric sequence ← here is an "r"

$t_1 = a = 81, 27, 9, 3, \ldots$

$r = \frac{t_2}{t_1}$ or $\left(\frac{t_3}{t_2}\right)$ or $\left(\frac{t_4}{t_3}\right)$

$= \frac{27}{81}$

$= \frac{1}{3}$

$t_n = ar^{n-1}$

$= 81\left(\left(\frac{1}{3}\right)^{n-1}\right)$

$t_{10} = 81\left(\frac{1}{3}\right)^{10-1}$

acceptable answer

$\doteq 0.00411$

preffered ans

$(3^4)(3^{-9})$

$= 3^{4+(-9)}$

$= 3^{-5}$

$= \frac{1}{3^5}$

$= \frac{1}{243}$

Example 7.2.3

From your text: Pg. 430 #8

Determine the recursive formula and the general term for the geometric sequence in which

a) the first term is 19 and the common ratio is 5

b) $t_1 = -9$ and $r = -4$

c) the first term is 144 and the second term is 36

d) $t_1 = 900$ and $r = \frac{1}{6}$

$t_n = ar^{n-1}$

-4^{n-1}

$(-4)^{n-1}$

a)

recursive	general term
$t_1 = 19$	$t_1 = (19)(5^{n-1})$
$t_n = (t_{n-1})5$	

b)

recursive	general term
$t_1 = -9$	$t_n = (-9)((-4)^{n-1})$
$t_n = (t_{n-1})(-4)$	

inside the bracket because we raise all of r to the exponent! "$n-1$"

$r = \frac{t_2}{t_1} = \frac{36}{144} = \frac{1}{4}$

c)

recursive	general
$t_1 = 144$	$t_n = (144)(\frac{1}{4})^{n-1}$
$t_n = (t_{n-1})(\frac{1}{4})$	

Example 7.2.4

Given a geometric sequence with $t_6 = -486$ and $t_9 = 13122$, determine the general term and the first 4 terms of the sequence.

↳ we need t_1 & r.

$t_6 = (a)r^5$

$t_9 = (a)r^8$

Note $\frac{t_9}{t_6} = \frac{\cancel{a}r^8}{\cancel{a}r^5}$

$\frac{13122}{-486} = r^3 \Rightarrow r = \left(\frac{13122}{-486}\right)^{\frac{1}{3}}$

$= (-27)^{\frac{1}{3}}$

$= -3$

$\boxed{\text{need } a}$

$t_6 = ar^5$

$\Rightarrow -486 = a(-3)^5$

$\Rightarrow a = \frac{-486}{(-3)^5}$

$= 2$

General Term ($t_n = ar^{n-1}$)

$t_n = (2)(-3)^{n-1}$

1st 4 terms

t_1, t_2, t_3, t_4

$2, -6, +18, -54$

Success Criteria:

- I can identify when a sequence is geometric, by seeing if it has a common ratio
- I can use the General Term Formula to develop an equation for an geometric sequence
- I can use the General Term to find any term in a sequence OR to find out how many terms are in a sequence
- I can recognize that an geometric sequence is always an exponential function