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MATH@TD

Unit 7 – Sequences and Series (Discrete Functions)

7.5 – Arithmetic Series

Learning Goal: We are learning to calculate the sums of the terms of an arithmetic sequence.

We have been studying sequences (which are ordered lists of numbers). We examined two types of sequences: Arithmetic and Geometric. We now turn our attention to a concept very closely related to sequences – Series.

Definition 7.5.1

A **Series** is constructed by **adding together the terms of a sequence**.

So an Arithmetic Series arises when we add together the terms of an Arithmetic Sequence.

Example 7.5.1

Given the 8 term arithmetic sequence $3, 7, 11, 15, 19, 23, 27, 31, \ldots$ determine the associated series. Determine the **PARTIAL SUM** S_4.

A partial sum occurs when you add up ***PART* of a series**

$S = 3 + 7 + 11 + 15 + 19 + 23 + 27 + 31$

$S_4 = 3 + 7 + 11 + 15 = \boxed{36}$

the 1st 4 terms

Obtaining a Partial Sum Formula

Karl Friedrich Gauss was really smart. He found a cool and quick way to add up the numbers from 1 to 100.

$$S_{100} = 1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100$$

$$+ \quad S_{100} = 100 + 99 + 98 + 97 + \ldots + 4 + 3 + 2 + 1$$

$$2S_{100} = 101 + 101 + 101 + 101 + \ldots + 101 + 101 + 101 + 101$$

$$\Rightarrow 2S_{100} = (100)(101) \Rightarrow S_{100} = \frac{(100)(101)}{2} = 5050$$

$a = 7$

$d = +3$

Consider now another generic arithmetic sequence. You tell me the first number, and the common difference. Let's generate 7 terms and add them up using Gauss's method.

$$S_7 = \overset{t_1}{7},\ 10,\ 13,\ 16,\ 19,\ 22,\ \overset{t_7}{25}$$

$$S_7 = \overset{t_7}{25},\ 22,\ 19,\ 16,\ 13,\ 10,\ \overset{t_1}{7}$$

$$2S_7 = 32 + 32 + 32 + 32 + 32 + 32 + 32$$

$$2S_7 = 7(32) \Rightarrow S_7 = \frac{7(32)}{2}$$

$t_1 + t_7$

First equation for an arithmetic series:

$$S_n = \frac{n(t_1 + t_n)}{2}$$

What happens if we don't know t_n? Is there a formula for t_n?

$$t_n = a + (n-1)d$$

Second equation for an arithmetic series:

$$S_n = \frac{n(\underbrace{t_1}_{=a} + a + (n-1)d)}{2}$$

$$\boxed{S_n = \frac{n(2a + (n-1)d)}{2}}$$

$$S_{20} = \frac{20(t_1 + t_{20})}{2}$$

$$S_{20} = \frac{20\left(2(5) - (19)(-3)\right)}{2}$$

Example 7.5.2

Determine the sum of the $\overbrace{\text{first 20 terms}}^{S_{20}}$ of the arithmetic sequence:

$5, 2, -1, -4, \ldots$

$t_1 = 5$

$d = t_2 - t_1$
$= 2 - 5$
$= -3$

Since we don't know t_{20}

$$S_n = \frac{n\left(2a + (n-1)d\right)}{2}$$

20 − 1

$$\Rightarrow S_{20} = \frac{20\left(2(5) + (19)(-3)\right)}{2} = -470$$

Example 7.5.3

From your text: Pg. 452 #5d

12th term in the associated sequence

$\boxed{t_n = a + (n-1)d}$

For the given arithmetic series determine t_{12} and S_{12}: $\frac{1}{5} + \frac{7}{10} + \frac{6}{5} + \frac{17}{10} + \ldots$

Arithmetic Sequence: $\frac{1}{5}, \frac{7}{10}, \frac{6}{5}, \frac{17}{10}, \ldots$

$a = t_1 = \frac{1}{5}$

$d = t_2 - t_1$
$= \frac{7}{10} - \frac{1}{5}$
$= \frac{7}{10} - \frac{2}{10}$
$= \frac{5}{10} = \frac{1}{2}$

$t_n = a + (n-1)d$

$t_{12} = \left(\frac{1}{5}\right) + (11)\left(\frac{1}{2}\right)$
$= \frac{1}{5} + \frac{11}{2}$
$= \frac{2}{10} + \frac{55}{10}$
$= \frac{57}{10}$

$$S_{12} = \frac{12(t_1 + t_{12})}{2}$$

$$S_{12} = \frac{12\left(2t_1 + (n-1)d\right)}{2}$$

Since we have t_1 & t_{12}, we use the 1st series formula

$$S_n = \frac{n(t_1 + t_{12})}{2}$$

$$\Rightarrow S_{12} = \frac{12\left(\frac{1}{5} + \frac{57}{10}\right)}{2} = \frac{177}{5}$$

$S_n = \frac{n(t_1 + t_n)}{2}$

we have t_1, t_n

we need "n" (the number of terms)

Example 7.5.4

From your text: Pg 452 #7e

Calculate the sum of the series: $-31 - 38 - 45 - \ldots - 136$

$t_1, t_2, t_3, \ldots, t_n$

t_{16}

$a = -31$

$d = -7$

$t_n = a + (n-1)d$

$-136 = -31 + (n-1)(-7)$

$\Rightarrow -105 = -7(n-1)$

$\div -7$

$15 = n - 1$

$\therefore \boxed{n = 16}$

$S_{16} = \frac{16(t_1 + t_{16})}{2}$

$= \frac{16(-31 + (-136))}{2}$

$= -1336.$

Success Criteria:

- I can calculate the sum of the first *n* terms of a arithmetic sequence by using one of the two formulas we learnt
  - $S_n = \frac{n[2a+(n-1)d]}{2}$
  - $S_n = \frac{n[t_1+t_n]}{2}$
- I can recognize when each formula is the most appropriate one to use