recall: sequence: 2, 6, 18, 54, 162, ...

series: 2 + 6 + 18 + 54 + 162 + ...

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# Unit 7 – Sequences and Series (Discrete Functions)

## 7.6 – Geometric Series

**Learning Goal:** We are learning to calculate the sum of the terms of a geometric sequence.

Again, a series is associated with a sequence. A series arises by adding together the terms of a sequence, so a Geometric Series arises by adding together the terms of a geometric sequence.

### The Partial Sum Formula for a Geometric Series

If we don't know the last term

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1, \qquad \text{OR} \qquad S_n = \frac{t_{n+1} - t_1}{r - 1}, \quad r \neq 1$$

if we know the last term $t_n$

$t_{n+1} = (t_n)r$

Remember that $r$ is the common ratio between successive terms!

$t_{n+1} = (t_n)(r)$

**Example 7.6.1**

sequence, consider the associated

Given the geometric series, and determine $t_7$ and $S_7$

sequence series

3, 12, 48, ...

$a = 3$

$r = \frac{t_2}{t_1} = \frac{12}{3} = 4$

$t_n = ar^{n-1}$

$\Rightarrow t_7 = ar^6$

$\Rightarrow t_7 = (3)(4^6) = 12,288$

Since $t_7 = 12\,288 \Rightarrow t_8 = (12288)(4) = 49\,152$

$S_n = \frac{t_{n+1} - t_1}{r - 1}$

$\Rightarrow S_7 = \frac{t_8 - t_1}{r - 1} = \frac{49152 - 3}{4 - 1} = 16,383$

**Example 7.6.2**

Determine $S_{10}$ for the geometric series 1.3, 3.25, 8.125, 20.3125,...

+ + +

using $S_n = \frac{a(r^n - 1)}{r - 1} \Rightarrow S_{10} = \frac{(1.3)(2.5^{10} - 1)}{2.5 - 1}$

$a = 1.3$

$r = \frac{t_2}{t_1} = \frac{3.25}{1.3} = 2.5$

$\doteq 8264.31$

$n = 10$

**Example 7.6.3**

seq: $t_1, t_2, t_3, t_4, \ldots, t_n$

Calculate the sum of the geometric series $2-6+18-54+\ldots+13122$

(we know the last term)

Given

$a = 2 = t_1$

$r = \frac{t_2}{t_1} = \frac{-6}{2} = -3$

$n = ?$

Use $S_n = \frac{t_{n+1} - t_1}{r-1}$

$t_{n+1} = (t_n)(r)$

$= (13\,122)(-3)$

$= -39\,366$

$\Rightarrow S_n = \frac{-39\,366 - 2}{-3-1}$

$= 9842$

Q. How many terms are there in the above series?

(i.e. what is "$n$")

$t_n = ar^{n-1}$

$\Rightarrow 13\,122 = (2)(-3)^{n-1}$

$\div 2$

$6561 = (-3)^{n-1}$

$(-3)^8 = (-3)^{n-1}$

$\Rightarrow n-1 = 8$

$\Rightarrow n = 9$

reasoning: Since $(-3)^{n-1} = +6561$

$\Rightarrow n-1$ is even

$(-3)^{n-1} = (+3)^{n-1}$

recall

$a^m = a^n$

$\Rightarrow m = n$

**Success Criteria:**

- I can add the first $n$ terms of a geometric sequence using:

$$S_n = \frac{a(r^n - 1)}{r-1}, \quad r \neq 1, \qquad \text{OR} \qquad S_n = \frac{t_{n+1} - t_1}{r-1}, \quad r \neq 1$$

- I can recognize when each formula is the most appropriate one to use