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MATH@TD

Unit 8 – Financial Mathematics

8.2 – Compound Interest and Future Value

Learning Goal: We are learning to determine the future value of a principal amount, using compound interest. This applies to both savings and loans.

Last day we look at the idea of Simple Interest (with formula $A = P(1+rt)$). Now we consider the notion of Compound Interest. Compound Interest is such that your savings grows much more quickly than it could if you were just earning simple interest.

Compound Interest Formulae:

Future Value

$$A = P(1+i)^n$$

this is just an exponential growth formula

where: $A =$ Total Amount of the investment/loan plus interest

$P =$ The Principal (amount invested/borrowed) — Present Value

$i =$ interest rate per compounding period — $i = \frac{r}{c}$ (annual interest rate; # of compounding periods in 1 year)

$n =$ the total number of compounding periods — $n = ct$ (# of compounding periods/yr.; # of years)

Now, there is one major challenge to any calculations. Interest is normally stated on a yearly basis, but interest is actually charged more frequently (the bank earns more money when you pay interest on interest ☹). So if you pay 12% interest per year, but pay interest monthly, you actually pay 1% each month. You must incorporate this into your calculations.

Here is a helpful chart:

	How often ~~you pay~~ interest is calculated (Times per year)	i value in your equation (with r = annual int. rate)	n value in your equation (with t years)
Annually	1	$i = \frac{r}{1}$	$n = (1)(t)$
Semi-annually	2	$i = \frac{r}{2}$	$n = (2)(t)$
Quarterly	4	$i = \frac{r}{4}$	$n = (4)(t)$
Monthly	12	$i = \frac{r}{12}$	$n = 12t$

Example 8.2.1

a) You deposit $10 000 into an account which pays 2.4% per year, compounded annually. What is the amount of money in the account in 10 years?

Given	Want
$P = 10\,000$	A
$i = \frac{0.024}{1}$	
$n = (1)(10)$	

$$A = P(1+i)^n$$
$$= 10000(1.024)^{10}$$
$$= \$12\,676.51$$

A nice sentence answering the question asked

b) You deposit $10 000 into an account which pays 2.4% per year, compounded monthly. What is the amount of money in the account in 10 years?

Given	Want
$P = 10000$	A
$i = \frac{0.024}{12}$	
$= 0.002$	
$n = (12)(10)$	
$= 120$	

$$A = P(1+i)^n$$
$$= 10\,000(1.002)^{120}$$
$$= \$12\,709.44$$ > compounding annually.

Example 8.2.2

Beth deposits $500 into an account which pays 6% compounded monthly. Fred deposits $500 into an account which pays 6% **simple interest**. What is the difference in the value of their accounts after 5 years?

Beth

Given	Want
$P = 500$	A
$i = \frac{0.06}{12}$	
$= 0.005$	
$n = (12)(5)$	
$= 60$	

$$A = P(1+i)^n$$
$$= 500(1.005)^{60}$$
$$= \$674.43$$

Fred

Given	Want
$P = 500$	A
$r = 0.06$	
$t = 5$	

$$A = P(1+rt)$$
$$= 500(1+(0.06)(5))$$
$$= \$650$$

∴ The difference is that compounding gives $24.43 extra dollars.

$674.43 - 650$

Example 8.2.3

On her 15th birthday, being very wise, Susan invests $10,000 in an account which pays 2.4% compounded monthly. The not-so-wise John waits until his 45th birthday to invest $10,000 in an account which pays 2.4% compounded monthly. How much is each account worth when they reach 65 years old?

Susan

Given	Want
$P = 10\,000$	A
$i = \frac{0.024}{12} = 0.002$	
$n = (12)(50) = 600$	

$A = P(1+i)^n$

$= 10000(1.002)^{600}$

$= \$33\,161.40$

John

Given	Want
$P = 10{,}000$	A
$i = \frac{0.024}{12} = 0.002$	
$n = (12)(20) = 240$	

$A = P(1+i)^n$

$= 10000(1.002)^{240}$

$= \$16\,153.$

∴ Susan has $33 161.40 while John $16 153 in their accounts at 65 years old.

Example 8.2.4

You find yourself in a furniture store. Looking around you become dazzled by the adverts promising a better and happier life if you only had one of their beautiful couches. The advertisement reads:

No Money Down!

No Payments for 3 years!!

Take this couch home today!!!!!

Only $1599*!!!!!!!!!!!!!

You decide it's a good deal, but you neglect to read the asterisk until it's too late. You've already signed on the dotted line. After signing you decide to finally read the fine print which says:

**Financed at 18% compounded monthly*

How much do you have to pay after 3 years?

How much of that is **just** interest?

Given	Want
$P = 1599$	A
$i = \frac{0.18}{12} = 0.015$	
$n = (12)(3) = 36$	

$$A = P(1+i)^n$$

$$= 1599(1.015)^{36}$$

$$= \$2732.91$$

$$I = A - P$$

$$= 2732.91 - 1599$$

$$= \$1133.91 \text{ in interest}$$

Success Criteria:

- I can determine the related interest (i) per compounding period (n)
- I can use the Future Value formula to solve various financial problems
- I can calculate the total interest earned/paid by calculating A – P.