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MATH@TD

Unit 8 – Financial Mathematics

8.4 – Annuities: Future Value

Learning Goal: We are learning how to determine the future value of an annuity earning compound interest.

The problem with the examples in 8.2 and 8.3 is that not many people have large amounts of money to be depositing into savings accounts. People usually make **regular deposits of smaller amounts of money**. **An account into which (or out of!) regular payments are made is called AN ANNUITY**. We will study two aspects of annuities: Future Value and Present Value.

Future Value of an Annuity

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

where: FV = Future Value n = total number of compounding periods

R = Regular Payment PV = Present Value.

i = interest rate per compounding period.

Example 8.4.1

Dylan decides to deposit $200 monthly into an account which pays 3.6% per year, compounded monthly. What is the value of his annuity after 25 years? How much interest is earned? If Dylan leaves the money in his account for another 20 years, but makes no more regular payments, how much money will be in the account at the end of 45 years?

Part ① Annuity

Given	What
$R = 200$	FV.
$i = \frac{0.036}{12} = 0.003$	
$n = (12)(25) = 300$	

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$= \frac{200\left((1.003)^{300} - 1\right)}{0.003}$$

$$= \$97\,086.10$$

$$I = A - P$$

$$= 97\,086.10 - (200)(300) = 37\,086.10$$

($60 000)

Part ② 2nd 20 years (compounded interest)

Given	What
$P = 97086.10$	A
$i = 0.003$	
$n = (12)(20) = 240$	

$$A = P(1+i)^n$$

$$= 97086.10(1.003^{240})$$

$$= \$199\,242.04$$

Example 8.4.2

Tingyi invests $300, every three months (i.e. quarterly) into an RRSP which pays (on average) ~~12~~ 4.8% compounded quarterly. What is the value of her RRSP after 45 years?

Given	Want
$R = 300$	FV
$i = \frac{0.048}{4} = 0.012$	
$n = (4)(45) = 180$	

$((300)(180) = \$54\,000)$

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$= \frac{300\left(1.012^{180} - 1\right)}{0.012}$$

$$= \$189,009.11$$

Example 8.4.3

An absolutely wonderful student decides to give Mr. Templeton $100,000 when he retires in ~~15~~ 5 years. The student finds an investment which pays 9% compounded monthly. What is the regular payment s/he would need to make to have $100,000 in ~~15~~ 5 years?

Given	Want
$FV = 100\,000$	R
$i = \frac{0.09}{12} = 0.0075$	
$n = (12)(5) = 60$	

$$FV = \frac{R[(1+i)^n - 1]}{i}$$ rearrange to isolate for R

$$(FV)(i) = R\left[(1+i)^n - 1\right]$$

$$\therefore \left[(1+i)^n - 1\right]$$

$$\Rightarrow R = \frac{(FV)(i)}{\left[(1+i)^n - 1\right]}$$

$$= \frac{(100000)(0.0075)}{\left(1.0075^{60} - 1\right)} = \$1325.84$$

Success Criteria:

- I can recognize that an annuity is a "regular payment" on a loan or investment
- I can use the Future Value Annuity formula to solve various financial problems
- I can calculate the total interest earned/paid by taking $I = FV - (n \times R)$.