

Unit 8 – Financial Mathematics

8.5 – Annuities: Present Value

Learning Goal: We are learning how to calculate the present value of an annuity earning compound interest.

Today we continue with the idea of an annuity, but we will be looking at **PRESENT VALUE (TODAY'S VALUE OF THE INVESTMENT/LOAN)**

Future Value of an Annuity

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

Present Value of an Annuity

$$PV = \frac{R[1 - (1+i)^{-n}]}{i}$$

The exponent is negative.

where: $FV =$ **Future Value**

$R =$ **The Regular Payment into or out of the annuity**

$i =$ **The interest rate per compounding period**

$n =$ **The total number of compounding periods over the life of the account**

$PV =$ **Present Value**

Example 8.5.1

While in college you want an **annuity** to pay you \$250 a month, every month over the four years you are studying. How much money would you need to invest today in an account which pays 2.4% compounded monthly to guarantee the monthly payment? What is the total amount of money you receive from the annuity? How much interest do you earn?

The total payment of the annuity

$$250 \times 12 \times 4 = \$12,000$$

$$PV = \frac{R[1 - (1+i)^{-n}]}{i}$$

$$= \frac{[250(1 - (1.002^{-48}))]}{0.002}$$

Given	Want
$R = 250$	PV
$i = \frac{0.024}{12} = 0.002$	(I)
$n = (12)(4) = 48$	

total payment

$$= \$11,431.11$$

$$I = A - PV$$

$$= 12,000 - 11,431.11 = \$568.89$$

Example 8.5.2

Henry borrows \$25,000 to buy a car. He pays \$3,000 down. He will take 6 years to pay off the loan. The bank charges 2.6% compounded biweekly. What are the regular payments Henry has to make to pay off the car? How much interest does he pay?

the car is \$28,000

Given
 $PV = 25000$
 $i = \frac{0.026}{26}$
 $= 0.001$
 $n = (26)(6)$
 $= 156$

Want
 R
 (I)
 $\Rightarrow [1 - (1+i)^{-n}]$

$$PV = \frac{R[1 - (1+i)^{-n}]}{i} \times i$$

$$\Rightarrow (PV)(i) = R[1 - (1+i)^{-n}]$$

$$R = \frac{(PV)(i)}{1 - (1+i)^{-n}}$$

how many times per year? biweekly means every 2 weeks
 $\Rightarrow 52$ wks/year biweekly
 \Rightarrow the # of compounding periods = $\frac{52}{2} = 26$

$$= \frac{(25000)(0.001)}{[1 - 1.001^{-156}]}$$

$$= \$173.16$$

\therefore Henry pay \$173.16 every two weeks

$$I = \text{Total paid} - \text{borrowed}$$

$$= (173.16)(156) - 25,000 = 2012.96$$

Success Criteria:

- I can recognize the difference between present value and future value
- I can use the Future Value Annuity formula to solve various financial problems
- I can calculate the total interest earned/paid by taking $I = (n \times R) - PV$.