Hak check pg 37

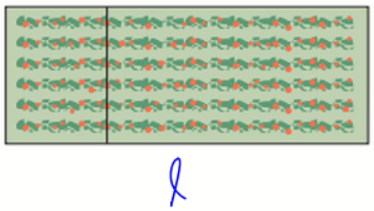
13. A farmer has 450 m of fencing to enclose a rectangular area and divide it into two sections as shown.

T

a) Write an equation to express the total area enclosed as a function of the width.

b) Determine the domain and range of this area function.

c) Determine the dimensions that give the maximum area.



a) $A = \ell \times w$ ("w")

$P = 3w + 2\ell$

$\Rightarrow 450 = 3w + 2\ell$

$\ell = \frac{450 - 3w}{2}$

$= 225 - \frac{3}{2}w$

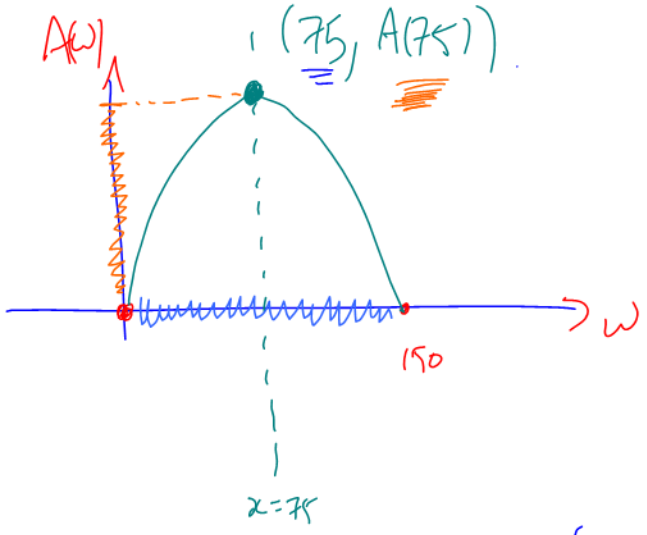
sub in A

$A(w) = \left(225 - \frac{3}{2}w\right)w$

quadratic

zeros form

b)



zeros form

$A(w) = \left(225 - \frac{3}{2}w\right)(w)$

$w = 0$

$\Rightarrow 225 - \frac{3}{2}w = 0$

$225 = \frac{3}{2}w$

$w = 225 \times \frac{2}{3}$

$= 150$ m.

$D_A = \{w \in \mathbb{R} \mid 0 \le w \le 150\}$

$A(75) = \left(225 - \frac{3}{2}(75)\right)(75)$

$= 8437.5\ m^2$

$R_A = \{A(w) \in \mathbb{R} \mid 0 \le A(w) \le 8437.5\ m^2\}$

c) Area max = $8437.5\ m^2$ when $w = 75$ m, and $\ell = 112.5$ m

Chapter 1 – Introduction to Functions

1.6 – 1.8: Transformations of Functions *(Part 1)*

To **TRANSFORM** something is to CHANGE (or MOVE)

Transformations of functions can be seen in two ways: algebraically, and graphically. We'll begin by examining transformations graphically.

But before we do, we need to remember that the graph of a function, $f(x)$, is given by:

$$f(x)=\left\{\left(x, f(x)\right) \mid x \in D_f\right\}$$

So, for functions we have two things (NUMBERS!) to "transform". We can apply transformations to

1) **Domain** values (which we call **HORIZONTAL TRANSFORMATIONS**)
2) **Range** values (which we call **VERTICAL TRANSFORMATIONS**)

There are **THREE BASIC FUNCTIONAL TRANSFORMATIONS**

1) Flips (*Reflections*)
2) Stretches (*Dilations*)
3) Shifts (*Translations*)

multiplication
addition

BEDMAS

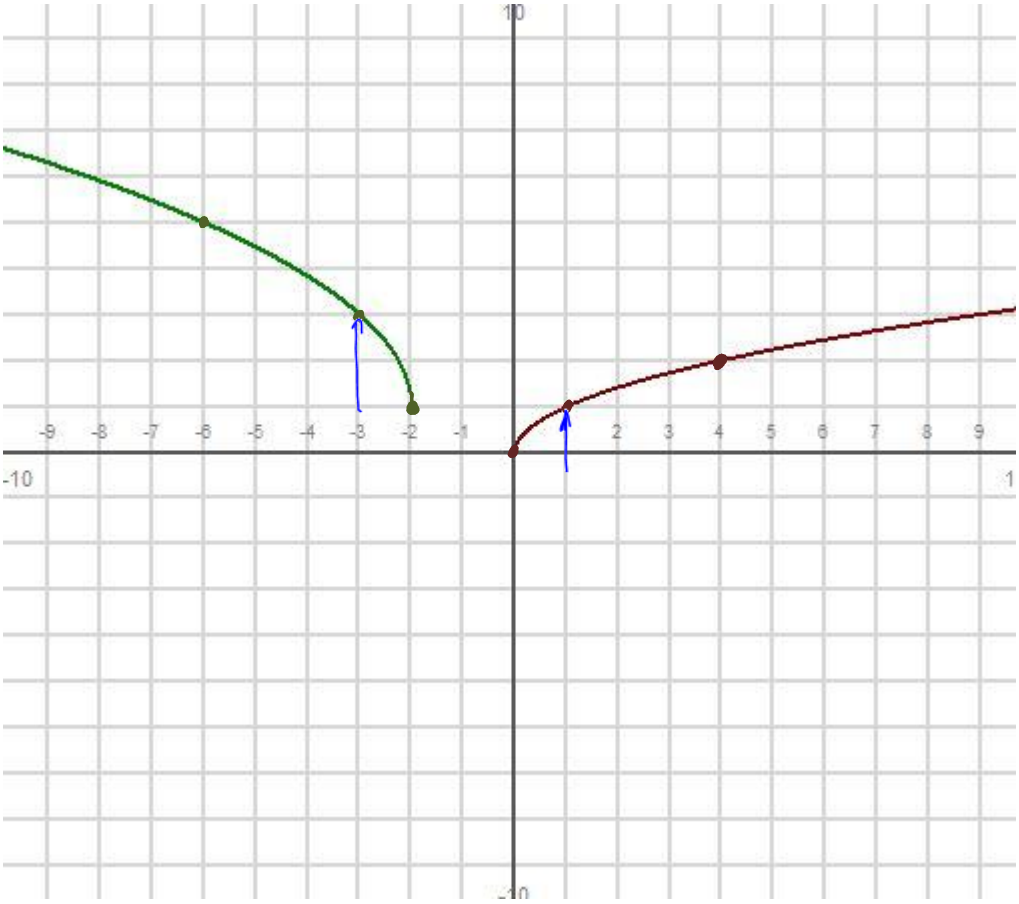
+, −, ×, ÷

So, we can have **Horizontal** flips, stretches and/or shifts, and **Vertical** flips, stretches and/or shifts. Now let's take a look at how transformations can be applied to functions.

***Note*:** We'll (mostly) be applying transformations to our so-called "parent functions" (although applying transformations to linear functions can seem pretty silly!)

Example 1.8.1

Consider, and make observations concerning the sketch of the graph of the parent function $f(x)=\sqrt{x}$ and the transformed function $g(x)=2\sqrt{-x-2}+1$.



Horizontal Transformations

Flip

SHIFT left by 2

Vertical Transformations

Stretch

Shift up 1

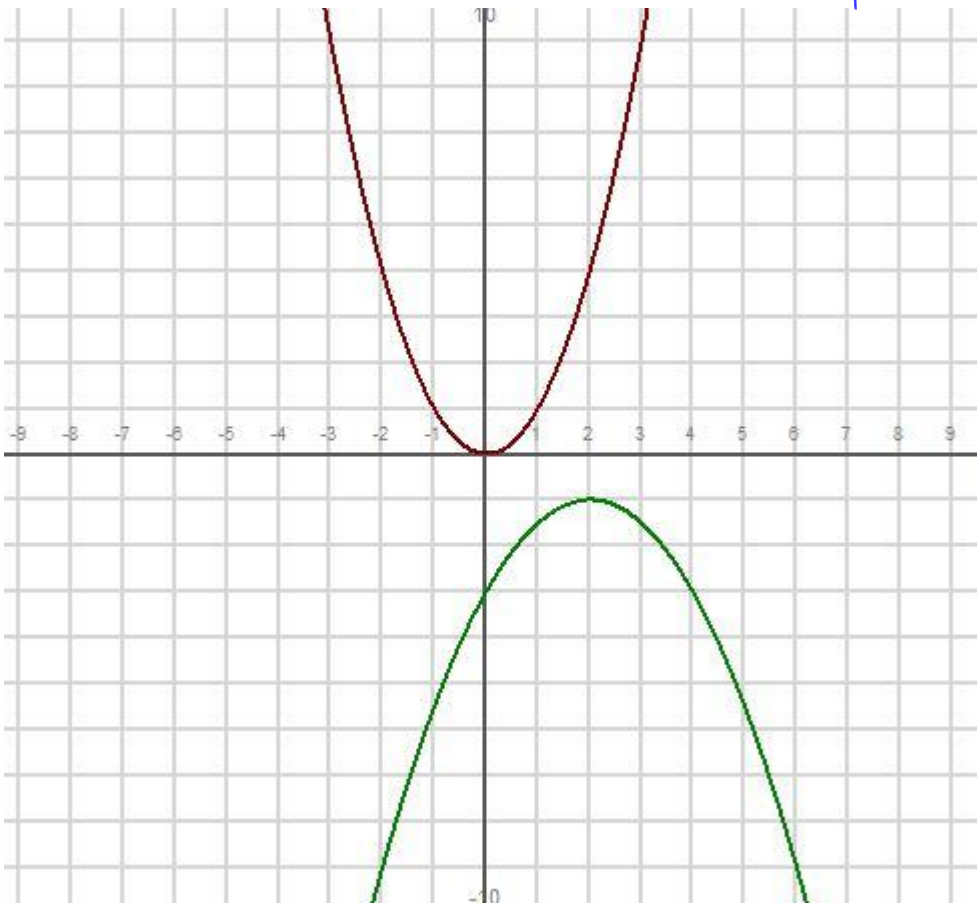
Note: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation $g(x)=2f(-x-2)+1$

Horizontal

Vertical

Example 1.8.2

Consider, and make observations concerning the sketch of the graph of the parent function $f(x)=x^2$ and the transformed function $g(x)=-\frac{1}{2}(x-2)^2-1$



Horizontal Transformations

Shift right 2

Vertical Transformations

Flip

Shift down 1

Note: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation

$g(x)=-\frac{1}{2}f(x-2)-1$

Chapter 1 – Introduction to Functions

1.6 – 1.8: Transformations of Functions *(Part 2)*

We now turn to examining Transformations of Functions from an algebraic point of view (although a geometric perspective will still shine though!)

Definition 1.8.1

Given a function $f(x)$ we can obtain a related function through functional transformations as

$$g(x)=af\left(k(x-d)\right)+c \text{ , where}$$

k must be "factored out"

a is the vertical stretch (dilation)
If $a<0$ we also have a vertical flip (reflection)

c is the vertical shift (translation)

k is the horizontal stretch with stretch factor $\frac{1}{k}$.
If $k<0$ we also have a horiz. flip.

d is the horizontal translation (shift)

Example 1.8.3

Consider the given function. State its parent function, and all transformations.

$$f(x)=3\sqrt{-x+2}-1 \quad = \quad 3\sqrt{-(x-2)}-1$$

Parent $g(x)=\sqrt{x}$

Horizontal Transformations

Flip Yes
Stretch ×1
Shift 2 right

Vertical Transformations

Flip No
Stretch ×3
Shift down 1

Example 1.8.4

The basic absolute value function $f(x)=|x|$ has the following transformations applied to it: Vertical Stretch -3, Vertical Shift 1 up, Horizontal Shift 5 right.
Determine the equation of the transformed function.

$g(x) = -3|x-5|+1$

Back to a geometric point of view

Sketching the graph of a transformed function can be relatively easy if we know:

1) The shape of the parent function AND a few (3 or 4) points on the parent.
2) How transformations affect the points on the parent
   i) Horizontal transformations affect the domain values (***OPPOSITE!!!!!!***)
   ii) Vertical transformations affect the range values

Note: Given a point on some parent function which has transformations applied to it is called an **IMAGE POINT** on transformed function.

Example 1.8.5

Given the sketch of the function $f(x)$ determine the image points of the transformed function $-2f\left(\frac{1}{3}(x+1)\right)+3$ and sketch the graph of the transformed function.

Vertical
Flip yes x-1
Stretch x2
Shift 3 up

Horizontal
Flip No
Stretch x 3
Shift 1 left

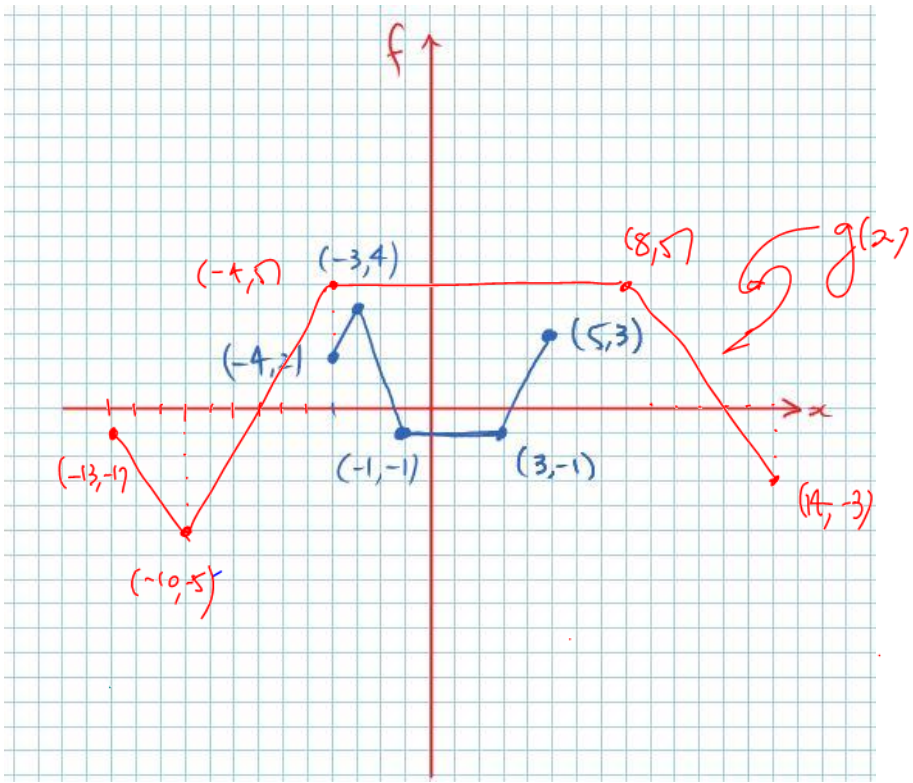
Parent

x	f
-4	2
-3	4
-1	-1
3	-1
5	3

Transformed

$x_T = 3x-1$ | $g = -2f+3$

-13	-1
-10	-5
-4	5
8	5
14	-3



Note: $(-4,2)$ is a point on the parent $(-13,-1)$ is the image of $(-4,2)$

Example 1.8.6

On the same set of axes sketch the graphs of $f(x)=\sqrt{x}$ and $g(x)=2\sqrt{-x+1}-2$.

Determine three points on the parent function and state the image points for each.

Vertical
Flip No
Stretch 2
Shift 2↓

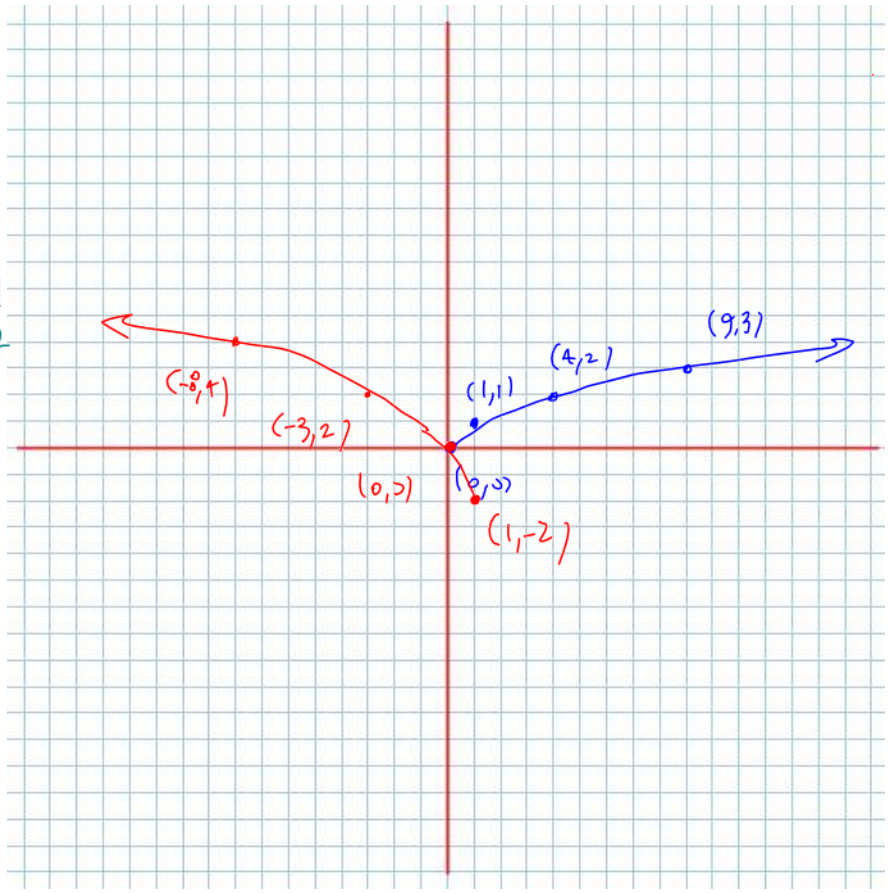
Horizontal
Flip Yes
Stretch 1
Shift 1→

Parent

x	$f=\sqrt{x}$
0	0
1	1
4	2
9	3

Transformed

$x_T=-x+1$	$g=2f-2$
1	−2
0	0
−3	2
−8	4



Class/Homework

Pg. 70 – 73 #4, 5b, 6, 7b, 8c, 9a, 10, 16, 17, 18, 19ac