Chapter 3 – Quadratic Functions

3.1 – Properties of Quadratic Functions

This lesson is a review of some of what we learned about quadratics in Grade 10. In Grade 10 we studied the **THREE FORMS** of quadratic functions and the **information** they give:

1) Standard Form - $f(x) = ax^2 + bx + c$

   Information

   y-int: $(0, f(0)) = (0, c)$ — if $a<0$ then the parabola opens "down"

   which way the parabola opens

2) Zeros (or Factored) Form - $f(x) = a(x-r)(x-s)$

   Information

   zeros: $x = r$, $x = s$

   which way the parabola opens! (same stretch factor "a")

   Axis of Symmetry: AoS: $x = \frac{r+s}{2}$

   vertex $\left(\frac{r+s}{2}, f\left(\frac{r+s}{2}\right)\right)$

3) Vertex Form - $f(x) = a(x-h)^2 + k$

   Information

   vertex (h, k)

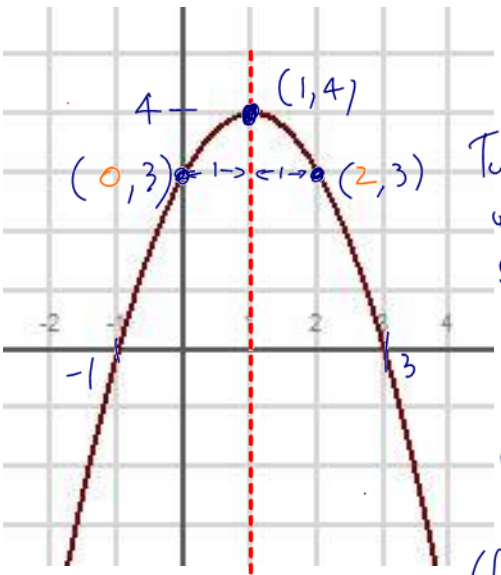
   which way the parabola opens

   AoS: $x = h$

   Max or min value: k.

Recall the concept of the axis of symmetry.

- vertical line passing through the vertex
- acts like a "mirror" for the two "halves" of a parabola
- The AoS is a line with eqn "$x = \#$"
- The AoS is the average of any two (domain) x-values which share the same functional value



Two points with the same y value give the AoS (find the avg. of the 2 domain values)

Example 3.1.1

Given the quadratic function $f(x)=\frac{1}{2}(x+3)^2-1$, state:

a) The direction the parabola opens
b) The coordinates of the vertex
c) The equation of the axis of symmetry

vertex form.

a) opens up: $a=\frac{1}{2}>0$

b) $(-3,-1)$

c) $x=-3$

vertex is a point containing info: the min value of -1 at $x=-3$

Example 3.1.2

Given the quadratic function $g(x)=-2(x+3)(x-1)$, state

a) The direction the parabola opens
b) The zeros of the quadratic
c) The equation of the axis of symmetry
d) The coordinates of the vertex
e) The function in vertex form

Sketch the graph of the function.

zeros form

$g(-1)=-2(-1+3)(-1-1)$
$=-2(2)(-2)$
$=8$

a) down: $a=-2<0$

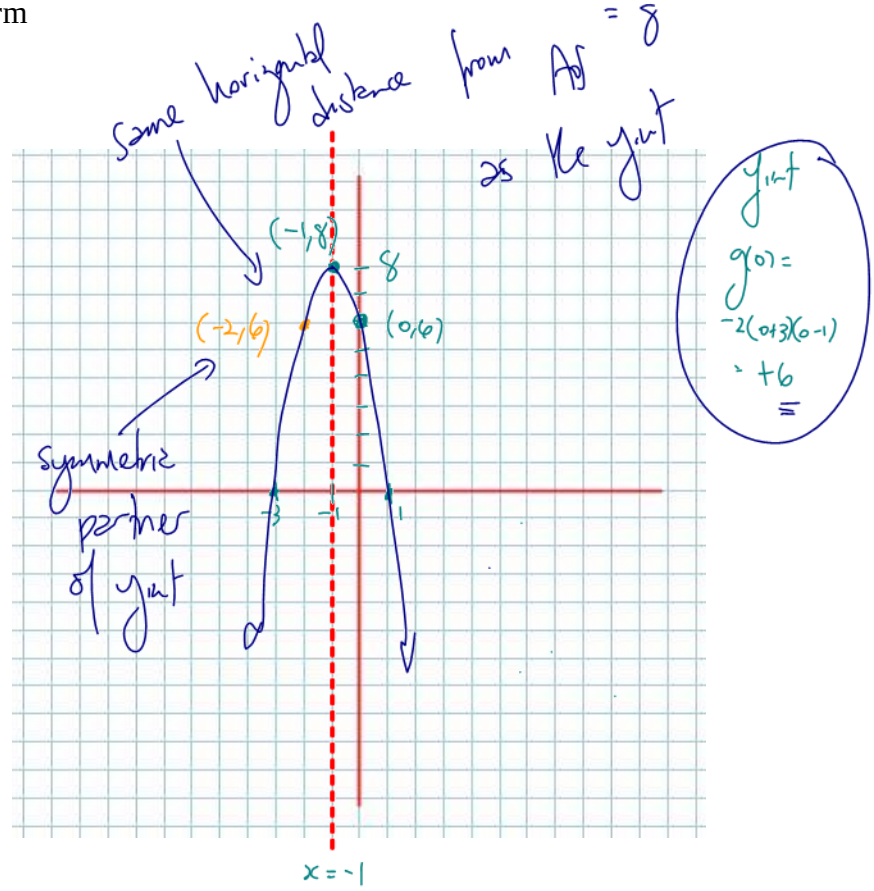
b) $x=-3,\ x=+1$

c) Aof: $x=\frac{-3+1}{2}=\frac{-2}{2}=-1$

$x=-1$

d) vertex $(-1, g(-1))$
$=(-1, 8)$

e) $g(x)=-2(x+1)^2+8$



Example 3.1.3

Given the two points $(4,7), (-5,7)$ which are on a parabola, determine the equation of the axis of symmetry.

same "y-value" These points are symmetric partners

$$\text{AoS}: \quad x = \frac{4 + (-5)}{2} = \frac{-1}{2}$$

Example 3.1.4 (From Pg. 147 in your text)

11. The height of a rocket above the ground is modelled by the quadratic function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

a) Graph the quadratic function.

b) How long will the rocket be in the air? How do you know? 8 seconds!

c) How high will the rocket be after 3 s?

d) What is the maximum height that the rocket will reach?

standard form

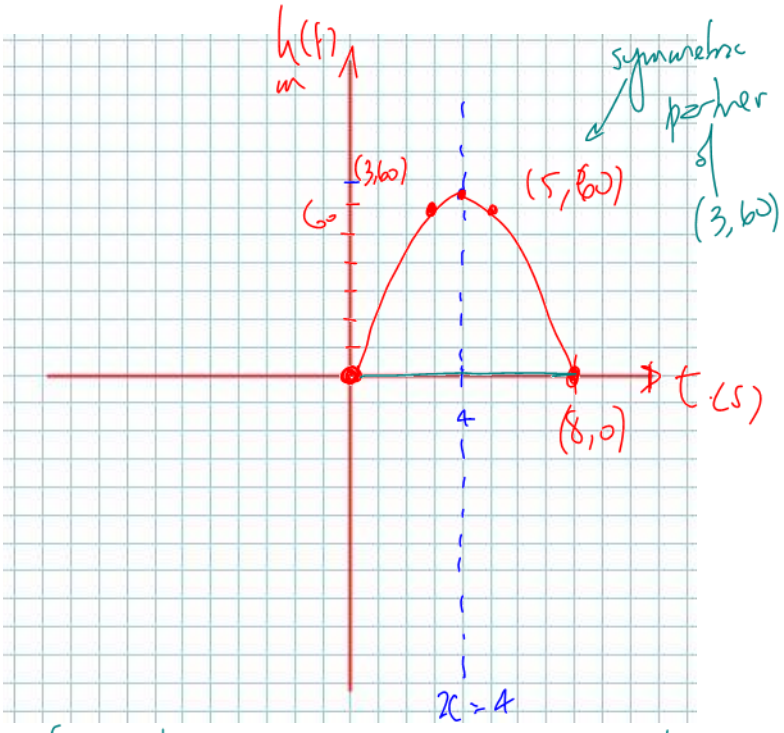
$h(t) = -4t^2 + 32t + 0$

y-int (0,0)

→ zeros form

$h(t) = -4t(t-8)$

zeros: $t=0, t=8$



c) $h(3) = -4(3)(3-8)$

$= -12(-5)$

$= 60$

$\text{AoS}: x = \frac{0+8}{2} = 4$

vertex $(4, h(4))$

$(4, 64)$

$D_h: \{t \in \mathbb{R} \mid 0 \le t \le 8\}$ $R_h: \{h(t) \in \mathbb{R} \mid 0 \le h(t) \le 64\}$

Class/Homework

Pg. 145 – 147 #3, 4, 6, 7, 8, 9de, 12 (tricky!)